MULTI-ITEM TWO-STAGE EOQ MODEL INCORPORATING QUANTITY AND FREIGHT DISCOUNTS

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Abstract: Supply chain management is an integrating function with primary responsibility for linking major business functions and business processes within and across companies into a cohesive and high-performing business models. Managing supply chain is a difficult task because of complex interrelations and integrations as various entities exist in it. One of the entities is transportation of items from source to destination. This process becomes tedious, when items are moving with one or more stoppage as on stoppage point inventory carrying cost would also be incurred. A single source, single destination and multi-item EOQ model has been discussed in the literature for single stage as well as two or more stages. Different discount policies are offered to procure and transport goods from the one stage to other stage. In the present study mathematical model is developed to minimize the sum of inventory, procurement and transportation costs when several items are shipped from a single source to single destination through an intermediate stoppage, in which it is assumed that inventory carrying charge at the intermediate stoppage, are very high after a pre-specified time. A solution procedure is proposed and verified with numerical illustration.

Keywords: Supply chain, Inventory management, Discount schemes, Transportation costs, Sequence of links

1. Introduction

The recent competitive pressures have made several organizations realize that individual companies by themselves cannot face up to the competition. A well designed product and a good technology for manufacture may not ultimately deliver value to the customer because of a bad set of suppliers or an inefficient distribution network. Every organization has three types of flows, the material flow, the information flow and the fund flow. While the material flows from the back end (supplier) of the supply chain to the front end (customer), money flows in the reverse direction. The information flows on both directions. Supply Chain Management (SCM) involves developing a set of management practices that will ensure that these three flows are smooth. Faster material flow will greatly improve responsiveness to customer requirements and will in turn ensure faster money flow back into the supply chain. Fast material flow is possible when different types of products are available under single roof, which are ordered in bulk by the buyer. This mean may be beneficial for the buyer to avail dual discount, one is quantity discount on the bulk purchase and other is freight discounts on weighted transportation. It is said that the ultimate goal of any effective supply chain management system is to reduce inventory; procure and transport the optimal quantity from source to destination, which becomes very important when cost is associated with all the activities discussed above. Same is the focus of current study to formulate an optimization problem to specifically determine the optimal order quantities in two stage supply chain.
along with the objective of minimizing the total cost that includes cost of purchasing, holding and transportation.

A variety of research papers have contributed significantly in the study of two stage supply chains, integrated discounts and transportation schemes among which few are as follows. Regarding the two stage supply chains, Kaminsky and Levi$^1$ in 2003 developed a two stage model of a manufacturing supply chain. This two stage production-transportation model featured capacitated production in two stages, and a fixed cost (or concave cost) for transporting the product between the stages. They show that their model reduces to a related model, with one capacitated production stage with linear production cost, and transportation between two inventory locations with non-linear transportation cost. However, Hyun-Soo Ahn and Philip Kaminsky$^2$ in 2005 considered a model of a two-stage push-pull production-distribution supply chain. In their model, orders arrive at the final stage according to a Poisson process. Two separate operations, which take place at different places with exponential service times, were located to convert the raw materials into finished goods. When the first operation is completed the intermediate inventory is held at the first stage and then transported to the second stage where the items are produced to order.

Subramanya and Sharma$^3$ in 2009 integrated two stage supply chain network of an automobile company, measured the performance parameters and established the priority decision and queuing rules for improving the utilization of resources. The study restricted to measuring operational processes in a two stage supply chain between the supplier - manufacturer - distributors. Hongwei Wang & Huixin Liu & Jian-bo Yang$^4$ in 2009 examined the dynamics of a two stage supply chain consisting of one retailer and one distributor with order-up-to control policy. Lee et al.$^5$ (2000) examined the value of information in a two-stage supply chain under an autoregressive demand process.

Rickst and Ventura$^6$ (2010) discussed two staged inventory models over an infinite planning horizon with constant demand rate and two modes of transportation. These transportation options include truckloads and a less than truck-load carrier. An optimal algorithm is derived for a one-warehouse one-retailer system. A power-of-two heuristic algorithm is also proposed for a one-warehouse multi-retailer system.

Many authors have also contributed their research in the field of integrating quantity discounts and distribution schemes. Hwang et al.$^7$ (1990) investigated the problem of determining optimal order quantity when all units’ quantity discounts are available on purchasing price and freight cost. Tersine and Barman$^8$, (1991) assumed a constant demand rate and developed a model with freight and price discounts, where freight discount structure is based on weight. Ertogral$^9$, (2008) took a single stage multi uncapacitated dynamic lot sizing problem (MILSP) with transportation cost and assumed finite planning horizon with dynamic demand. He considered all unit inventory management models to formulate the problem with piece wise linear transportation cost function. Mendoza and Ventura$^{10}$, (2008) developed an unconstrained integrated inventory-transportation model to decide optimal order quantity for inventory system over a finite horizon.

In this research paper an integrated inventory-transportation two stage supply chain model has been discussed incorporating the discounted policies on purchasing goods and transportation network. A model is formulated which explains the flow of ordered quantity from single source to single destination with one intermediate stoppage point. In the model buyer at destination avails quantity discounts on bulk order and freight discounts on bulk transported quantity. Quantity discounts are provided by the supplier, in which supplier has fixed the quantity level beyond which discount would be given. The mode of transporting the goods from source to destination takes place in two stages, where in the first stage, goods are moved using cargo. The unloading point of goods is the intermediate stoppage. At the stoppage, unloading of goods and their further processing takes a specified time for which the holding cost is free.
Since cargo points require space to unload goods of number of cargos, the holding cost at stoppage increases with very high rates after a preset time. Keeping inventory for long time may not be beneficial for the buyer but sometimes the inventory has to be kept because of some undue cause like transportation facility is not available to move goods from cargo point to destination. As the buyer has received discounts from supplier on the ordered quantity mentioned above, Cargo Company is giving discounts on weighted quantity from supply point to intermediate stoppage. Movement of goods from intermediate stoppage to destination is the second stage of model which is completed through modes of transportation, which are categorized as truck load (TL) and less than truck load (LTL) transportation. In Truck load transportation, the cost is a fixed of one truck up to a given capacity. In this mode company may use less than the capacity available but cost per truck will not be deducted. However in some cases the weighted quantity may not be large enough to substantiate the cost associated with a TL mode. In such situation, a LTL mode may be used. LTL may be defined as a shipment of weighted quantity which does not fill a truck. In such case transportation cost is taken on the bases of per unit weight. The model shows some ending inventory at destination to fulfill the uncertainties, as shortages are not allowed at any cost. The formulation and solution of the above enlightened model has been discussed in the following sections.

2. Model Sets, and Symbols
The formulated mathematical model is based on the following sets, and symbols.

2.1 Sets
• Product set with cardinality $P$ and indexed by $i$.
• Period set with cardinality $T$ and indexed by $t$.
• Item discount break point set with cardinality $L$ and indexed by small $l$.
• Freight discount break point set with cardinality $K$ and indexed by small $k$.
• Waiting time set at intermediate stoppage with cardinality $\Gamma$ and indexed by small $\tau$.

2.2 Parameters
$C$  Total cost
$c_{it}$  Cost of unit weighted quantity of period $t$
$D_{it}$  Demand for item $i$ in period $t$
$O^f_i$  Holding cost incurred on the total weight in period $t$ for $\tau$ days
$h_i$  Inventory holding cost per unit of item $i$
$w_i$  Per unit weight of item $i$
$\phi_{it}$  Unit purchase cost for $i^{th}$ item in $t^{th}$ period
$\beta_t$  Fixed freight cost for each truck load in period $t$
$\omega$  Weight transported in each full truck(in kgs)
$s$  Cost/kg of transportation in LTL policy
$a_{ilt}$  Limit beyond which a price break becomes valid in period $t$ for item $i$ for $l^{th}$ price break
$b_{kt}$  Limit beyond which a freight break becomes valid at period $t$ for $k^{th}$ price break
$d_{ilt}$  It reflects the fraction of regular price that the buyer pays for purchased items.
$f_{kt}$  It reflects the fraction of regular price that the buyer pays for transported weights.
\( IN_i \) Inventory level at the beginning of planning horizon for product \( i \)

### 2.3 Decision variables

- \( X_{it} \) Amount of item \( i \) ordered in period \( t \)
- \( R_{ilt} \) If the ordered quantity falls in \( l^{th} \) price break then the variable takes value 1 otherwise zero

\[
R_{ilt} = \begin{cases} 
1 & \text{if } X_{it} \text{ falls in } l^{th} \text{ pricebreak} \\
0 & \text{otherwise} 
\end{cases}
\]

- \( I_{it} \) Inventory level at the end of period \( t \) for product \( i \)
- \( j_t \) Total number of truck loads in period \( t \)
- \( y_t \) Amount in excess of truckload capacity (in weights)
- \( u_t \) The variable \( u_t \) (or, \( 1- u_t \)) reflects usage of policies, either TL and LTL policies or only TL policy i.e.

\[
u_t = \begin{cases} 
1 & \text{if considering TL & LTL both policies} \\
0 & \text{if considering only TL policy} 
\end{cases}
\]

- \( L_{2t} \) total weighted quantity transported in stage 2 of period \( t \)
- \( L_{1t} \) total weighted quantity transported in stage 1 of period \( t \)
- \( Z_{kt} \) If the weighted quantity transported falls in \( k^{th} \) price break then the variable takes value 1 otherwise zero

\[
Z_{kt} = \begin{cases} 
1 & \text{if } L_t \text{ falls in } k \text{ pricebreak} \\
0 & \text{otherwise} 
\end{cases}
\]

- \( v_{\tau t} \)

\[
\begin{cases} 
1 & \text{if } L_{\tau} \text{ waits at halt} \\
0 & \text{otherwise} 
\end{cases}
\]

### 3. Mathematical Model

The optimization problem minimizing the cost of purchasing, holding and transportation subject to procurement and distribution constraints is formulated as follows.
3.1 Model Formulation

Minimize
\[ C = \sum_{t=1}^{T} \left[ \left( \sum_{l=1}^{P} \left( \sum_{i=1}^{L} R_{ilt} d_{ilt} \phi_{ilt} X_{it} \right) \right) + \sum_{k=1}^{K} Z_{kt} \delta_{kt} \xi_{il} L_{lt} + \sum_{t=1}^{T+1} L_{it} \right] \]
\[ + \sum_{t=2}^{T+1} \left( s_{yt} + j_{l} \beta_{l} \right) u_{t} + \left( j_{l} + 1 \right) \beta_{l} \left( 1 - u_{t} \right) + \sum_{t=2}^{T+1} \sum_{i=1}^{P} h_{i} I_{it} \]

subject to
\[ I_{it} = I_{it-1} + X_{it} - D_{it} \quad \text{where} \quad i = 1, \ldots, P; \quad t = 2, \ldots, T \]
\[ I_{i1} = iN_{i1} + X_{i1} - D_{i1} \quad \text{where} \quad i = 1, \ldots, P \]
\[ \sum_{t=1}^{T} I_{it} + \sum_{t=1}^{T} X_{it} \geq \sum_{t=1}^{T} D_{it} \quad \text{where} \quad i = 1, \ldots, P \]
\[ X_{it} \geq \sum_{l=1}^{L} a_{ilt} R_{ilt} \quad i = 1, \ldots, P; \quad t = 1, \ldots, T \]
\[ \sum_{l=1}^{L} R_{ilt} = 1 \quad i = 1, \ldots, P; \quad t = 1, \ldots, T \]
\[ L_{it} = \sum_{i=1}^{P} w_{i} X_{it} \quad t = 1, \ldots, T \]
\[ L_{lt} \geq \sum_{k=1}^{K} b_{kt} Z_{kt} \quad t = 1, \ldots, T \]
\[ \sum_{k=1}^{K} Z_{kt} = 1 \quad t = 1, \ldots, T \]
\[ L_{lt} = L_{2t+1} \quad t = 1, \ldots, T \]
\[ L_{2t} \leq \left( y_{t} + j_{l} \omega \right) u_{t} + \left( j_{l} + 1 \right) \omega \left( 1 - u_{t} \right) \quad t = 2, \ldots, T + 1 \]
\[ L_{2t} = \left( y_{t} + j_{l} \omega \right) \quad t = 2, \ldots, T + 1 \]
\[ X_{it}, L_{1t}, L_{2t} \geq 0, R_{ilt} = 0 \text{ or } 1, Z_{kt} = 0 \text{ or } 1, v_{tt} = 0 \text{ or } 1, u_{t} = 0 \text{ or } 1 \]

\( l_{it}, y_{t}, j_{t} \) are integers, \( i=1,\ldots,P, \ t=1,\ldots,T, \ l=1,\ldots,L \) and \( k=1,\ldots,K. \)

3.2 Analysis of the Formulation

Objective function

The objective of the optimization problem is to minimize the cost incurred in purchasing the items reflected by the first term of the objective function; transportation cost from the source to the intermediate stoppage reflected by the second term; holding cost at the intermediate stoppage of the weighted quantity reflected by the third term; transportation cost from intermediate stoppage to the final destination reflected by the fourth term; and the last term reflects the ending inventory carrying cost at the destination. The cost is calculated for the duration of the planning horizon. The ordering cost is a fixed cost not affected by the ordering quantities and therefore is not the part of objective function.

\[
\begin{align*}
\text{Min } C &= \sum_{t=1}^{T} \left[ \sum_{i=1}^{P} \left( \sum_{l=1}^{L} R_{ilt} d_{ilt} \phi_{lt} X_{it} \right) + \sum_{k=1}^{K} Z_{kt} f_{kt} c_{lt} L_{lt} + \sum_{\tau=1}^{T} L_{lt} O_{t}^{\tau} v_{tt} \right] \\
&+ \sum_{t=2}^{T+1} \left( s_{yt} + j_{t} \beta_{l} \right) u_{t} + (j_{t} + 1) \beta_{l} (1-u_{t}) + \sum_{t=2}^{T+1} \sum_{i=1}^{P} h_{t} I_{it}
\end{align*}
\]

Constraints

The inventory level at period \( t \) is dependent upon inventory left in the last period, the quantity \( X_{it} \) ordered at period \( t \) and at demand \( D_{it} \) in the following way:

\[
I_{it} = I_{it-1} + X_{it} - D_{it}, \quad \text{where } i=1,\ldots,P, \ t=2,\ldots,T
\]

The inventory level at the end of the first period for item type \( i \) is composed of the inventory level at the beginning of the planning horizon, and the net change at the end of period one,

\[
I_{i1} = IN_{i1} + X_{i1} - D_{i1} \quad \text{where } i=1,\ldots,P
\]

The sum of ending inventory and optimal order quantity is more than the demand of all the periods, i.e.

\[
\sum_{t=1}^{T} I_{it} + \sum_{t=1}^{T} X_{it} \geq \sum_{t=1}^{T} D_{it}, \quad i=1,\ldots,P
\]

The buyer will order minimum quantity \( a_{ilt} \) to get discount i.e.
\[ X_{it} \geq \sum_{l=1}^{L} a_{ilt} R_{ilt} \quad i = 1, \ldots, P \quad t = 1, \ldots, T \]

It shows that the order quantity of all items in period \( t \) exceeds the price break threshold.

In any period, exactly one level will be activated either discount or no discount situation therefore,

\[ \sum_{l=1}^{L} R_{ilt} = 1 \quad i = 1 \ldots P, \quad t = 1 \ldots T \]

Transported quantity according to item weight is:

\[ L_{it} = \sum_{i=1}^{P} w_{i} X_{it} \quad t = 1, \ldots, T \]

Above constraint is an integrated constraint for procurement and distribution.

The minimum quantity transported is \( b_{it} \) i.e.

\[ L_{1t} \geq \sum_{k=1}^{K} b_{kt} Z_{kt} \quad t = 1, \ldots, T \]

It shows that the transported quantity of all items in period \( t \) exceeds the freight break threshold.

In any period, exactly one level will be activated depending on the weighted quantity transported

\[ \sum_{k=1}^{K} Z_{kt} = 1 \quad t = 1, \ldots, T \]

The total weighted quantity transported in stage 1 of period \( t \) is equal to the total weighted quantity transported in stage 2 of period \( t + 1 \)

\[ L_{1t} = L_{2t+1} \quad t = 1, \ldots, T \]

The minimum weighted quantity transported is equal to:

\[ L_{2t} \leq (y_t + j_t \omega) u_t + \omega (j_t + 1) (1 - u_t) \quad t = 2, \ldots, T + 1 \]

Overhead units from truckload capacity in weights are:

\[ L_{2t} = (y_t + j_t \omega) \quad t = 2, \ldots, T + 1 \]
**Price Breaks**

As discussed above, variable $R_{ilt}$ specifies the fact that when the order size at period $t$ is larger than $a_{ilt}$ it results in discounted prices for the ordered items for which the price breaks are defined as:

**Price breaks for ordering quantity are:**

$$d_f = \begin{cases} 
  d_{ilt} & a_{ilt} \leq X_{ilt} \leq a_{il,t+1} \\
  d_{il,t} & X_{ilt} \geq a_{ilt} 
\end{cases}$$

$i = 1, \ldots, P; \quad t = 1, \ldots, T; \quad l = 1, \ldots, L$

**Freight breaks for transporting quantity are:**

$$d_f = \begin{cases} 
  f_{kt} & b_{kt} \leq L_t \leq b_{k,t+1} \\
  f_{kl,t} & L_t \geq b_{Kt} 
\end{cases}$$

$i = 1, \ldots, T$

where $b_{kt}$ is the minimum required quantity to be transported in cargo.

4. **Numerical Illustration**

The model is validated for 3 products (Digital camera, LCD, Laptop), 3 periods (period size is of 1 week) to find the optimal and transported quantity for which the assumed data and solution are as follows.

4.1 **Data sets**

**Quantity Demanded ($D_{it}$)**

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digital Camera</td>
<td>75</td>
<td>65</td>
<td>95</td>
</tr>
<tr>
<td>LCD</td>
<td>505</td>
<td>200</td>
<td>70</td>
</tr>
<tr>
<td>Laptop</td>
<td>60</td>
<td>170</td>
<td>550</td>
</tr>
</tbody>
</table>

**Unit Cost (in $) per product per period ($Φ_{it}$)**

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digital Camera</td>
<td>250</td>
<td>240</td>
<td>230</td>
</tr>
<tr>
<td>LCD</td>
<td>1290</td>
<td>1280</td>
<td>1270</td>
</tr>
<tr>
<td>Laptop</td>
<td>645</td>
<td>535</td>
<td>725</td>
</tr>
</tbody>
</table>
### Maximum Initial inventory in the starting of planning horizon ($IN_i$)

<table>
<thead>
<tr>
<th>Product</th>
<th>Digital Camera</th>
<th>LCD</th>
<th>Laptop</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
<td>256</td>
<td>30</td>
</tr>
</tbody>
</table>

### Transportation cost (in $) of weighted quantity from source to intermediate stoppage ($c_i$)

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

### Weight per product ($w_i$) per kg

<table>
<thead>
<tr>
<th>Product</th>
<th>Digital Camera</th>
<th>LCD</th>
<th>Laptop</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>3</td>
</tr>
</tbody>
</table>

### Holding cost (in $) at intermediate stoppage ($o_i^r$)

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

### Waiting time at intermediate stoppage in days ($\tau$)

<table>
<thead>
<tr>
<th>$v_{it}$</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>Free</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

### Per Truckload capacity ($\omega$)=1000; Cost per kg of transporting in LTL policy $s$= $12

### Fixed freight cost (in $) for each truckload from intermediate stoppage to destination ($\beta_i$)

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1000</td>
<td>1050</td>
<td>1100</td>
</tr>
</tbody>
</table>
Holding Cost (in $) per product at destination ($h_i$)

<table>
<thead>
<tr>
<th>Digital Camera</th>
<th>LCD</th>
<th>Laptop</th>
</tr>
</thead>
<tbody>
<tr>
<td>34</td>
<td>148</td>
<td>84</td>
</tr>
</tbody>
</table>

Quantity thresholds ($a_{il}$) and discount factors ($d_{il}$) of Digital Camera

<table>
<thead>
<tr>
<th>Quantity Thresholds</th>
<th>Discount Factor Period 1</th>
<th>Quantity Thresholds</th>
<th>Discount Factor Period 2</th>
<th>Quantity Thresholds</th>
<th>Discount Factor Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq X_{it} &lt; 70$</td>
<td>1</td>
<td>$0 \leq X_{it} &lt; 75$</td>
<td>1</td>
<td>$0 \leq X_{it} &lt; 80$</td>
<td>1</td>
</tr>
<tr>
<td>$70 \leq X_{it} &lt; 80$</td>
<td>0.98</td>
<td>$75 \leq X_{it} &lt; 85$</td>
<td>0.89</td>
<td>$80 \leq X_{it} &lt; 90$</td>
<td>0.92</td>
</tr>
<tr>
<td>$80 \leq X_{it} &lt; 90$</td>
<td>0.96</td>
<td>$85 \leq X_{it} &lt; 95$</td>
<td>0.85</td>
<td>$90 \leq X_{it} &lt; 100$</td>
<td>0.87</td>
</tr>
<tr>
<td>$90 \leq X_{it}$</td>
<td>0.94</td>
<td>$95 \leq X_{it}$</td>
<td>0.80</td>
<td>$100 \leq X_{it}$</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Quantity threshold ($a_{il}$) of LCD

<table>
<thead>
<tr>
<th>Quantity Thresholds</th>
<th>Discount Factor Period 1</th>
<th>Quantity Thresholds</th>
<th>Discount Factor Period 2</th>
<th>Quantity Thresholds</th>
<th>Discount Factor Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq X_{il} &lt; 105$</td>
<td>1</td>
<td>$0 \leq X_{il} &lt; 115$</td>
<td>1</td>
<td>$0 \leq X_{il} &lt; 120$</td>
<td>1</td>
</tr>
<tr>
<td>$105 \leq X_{il} &lt; 205$</td>
<td>0.95</td>
<td>$115 \leq X_{il} &lt; 215$</td>
<td>0.85</td>
<td>$120 \leq X_{il} &lt; 240$</td>
<td>0.79</td>
</tr>
<tr>
<td>$205 \leq X_{il} &lt; 305$</td>
<td>0.90</td>
<td>$215 \leq X_{il} &lt; 315$</td>
<td>0.82</td>
<td>$240 \leq X_{il} &lt; 480$</td>
<td>0.72</td>
</tr>
<tr>
<td>$305 \leq X_{il}$</td>
<td>0.85</td>
<td>$315 \leq X_{il}$</td>
<td>0.75</td>
<td>$480 \leq X_{il}$</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Quantity threshold ($a_{il}$) of Laptop

<table>
<thead>
<tr>
<th>Quantity Thresholds</th>
<th>Discount Factor Period 1</th>
<th>Quantity Thresholds</th>
<th>Discount Factor Period 2</th>
<th>Quantity Thresholds</th>
<th>Discount Factor Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq X_{il} &lt; 95$</td>
<td>1</td>
<td>$0 \leq X_{il} &lt; 100$</td>
<td>1</td>
<td>$0 \leq X_{il} &lt; 200$</td>
<td>1</td>
</tr>
<tr>
<td>$95 \leq X_{il} &lt; 99$</td>
<td>0.75</td>
<td>$100 \leq X_{il} &lt; 200$</td>
<td>0.87</td>
<td>$200 \leq X_{il} &lt; 4000$</td>
<td>0.82</td>
</tr>
</tbody>
</table>
4.2 Solution

The Ordered Quantity ($X_{it}$) of given products in the respective periods are:

<table>
<thead>
<tr>
<th>Product</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digital Camera</td>
<td>70</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>LCD</td>
<td>411</td>
<td>116</td>
<td>0</td>
</tr>
<tr>
<td>Laptop</td>
<td>30</td>
<td>720</td>
<td>0</td>
</tr>
</tbody>
</table>

Discounts availed by buyer on ordered quantity:

<table>
<thead>
<tr>
<th>Product</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digital Camera</td>
<td>2%</td>
<td>0%</td>
<td>18%</td>
</tr>
<tr>
<td>LCD</td>
<td>15%</td>
<td>15%</td>
<td>0%</td>
</tr>
<tr>
<td>Laptop</td>
<td>0%</td>
<td>22%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Transported weights in both the stages ($L_{it}$ & $L_{2t+1}$)

<table>
<thead>
<tr>
<th>$L_{1t}$ = $L_{21}$</th>
<th>$L_{12}$ = $L_{22}$</th>
<th>$L_{13}$ = $L_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4340</td>
<td>3360</td>
<td>325</td>
</tr>
</tbody>
</table>
Discounts availed by buyer on weight quantity:

<table>
<thead>
<tr>
<th>Period</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>8%</td>
<td>6%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Number of days weighted quantity waited at intermediate stoppage ($\nu_t$):

<table>
<thead>
<tr>
<th>Waiting days</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Policy used for truckload schemes ($u_t$):

<table>
<thead>
<tr>
<th>Period</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL</td>
<td>TL</td>
<td>TL</td>
</tr>
</tbody>
</table>

Number of trucks used for transportation ($j_t$):

<table>
<thead>
<tr>
<th>Period</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Overhead units ($y_t$):

<table>
<thead>
<tr>
<th>Period</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>340</td>
<td>360</td>
<td>325</td>
</tr>
</tbody>
</table>

Ending Inventory at destination ($I_{it}$):

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digital Camera</td>
<td>45</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>LCD</td>
<td>162</td>
<td>78</td>
<td>8</td>
</tr>
<tr>
<td>Laptop</td>
<td>0</td>
<td>550</td>
<td>0</td>
</tr>
</tbody>
</table>

5. Conclusions

In this paper we have investigated the two stage supply chain optimization model that minimizes purchasing, holding and transportation cost. The ordered quantity is transported from the single source to
single destination through an intermediate stoppage at which the mode of transportation is changed. The different discount schemes are given by the supplier on the ordered quantity and by the transporter on the weighted quantity to the buyer. Finally the optimal ordered quantity, holding inventory and weighted transported quantity are determined. Hence we can conclude from our present research that integration of various functions of different entities is possible, in order to minimize the aggregate cost of purchasing and transportation activities. In fact the results of this study open several opportunities for further research and improvements. In future research, different extensions to the proposed models can be considered. Although the focus has been on single-stage models, we believe that these inventory models provide a strong foundation for subsequent analyses of multi-stage systems, since previous efforts in quantity discounts have primarily been focused on single-stage models involving only suppliers and producers.

References