Relativistic determinism – the clash with logic

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Consider the theory of special relativity (SR) as formalized in classical first-order predicate logic (FOPL). In this paper, we wish to argue that the phenomenon of relativistic determinism, which follows from the Lorentz transformations and the relativity of simultaneity, makes SR intolerant of undecidable propositions and hence logically inconsistent.

Let A and B be relatively moving inertial observers who happen to coincide in space at a given instant defined by $t = 0$ in A's frame and $t' = 0$ in B's frame. Let C be an event that is localized in spacetime and is distant to both A and B. Let $U(IBC)$ define a non-trivial universe of material objects with certain well-posed initial-boundary conditions IBC. Define the proposition P as "From A's point of view, C occurs in $U(IBC)$ when A's local clock reads $t = 0$" and the proposition Q as "From B's point of view, C occurs in $U(IBC)$ when B's local clock reads $t' = T$". Here $T > 0$ is a constant obtained from the Lorentz transformations as applied to the event C in A's and B's inertial frames. For clarity, information about spatial locations of C is suppressed from these propositions. Relativistic determinism asserts that if P is true then Q must be true (or $P \Rightarrow Q$); in other words, B's future at time $t' = 0$ is determined by the fact that A has observed C at precisely that instant (when A and B coincided) and so B must necessarily observe C at $t' = T$.

In order to obtain a logical contradiction from the above scenario, let us further stipulate that the proposition "Event C occurs in $U(IBC)$" is undecidable in SR, i.e., in particular, neither A nor B can either prove or refute this proposition. Such undecidability could occur in many ways, for example, as a result of Gödel's incompleteness theorems; alternatively, C could be a probabilistic event, such as, the outcome of a coin toss experiment or some quantum phenomenon; or else, C could be completely unpredictable as a result of being decided by the instantaneous free will of a human being. It immediately follows that P and Q are undecidable in SR; see the ensuing paragraph for the definition of such undecidability. Note that SR requires $P \Leftrightarrow Q$ to be a theorem.

Henceforth, whenever we refer to A (B), it is to be understood that our argument may apply equally well to any observer in A's (B's) set of inertial frames. Note that we require the following restrictions regarding propositions involving P and Q. The truth of P (Q) can be *asserted* (via an observation, for example) or *deduced* in SR *only* by A (B). However, B (A) can consider and either accept or refute in SR any assertion/deduction of the truth of P (Q) made by A (B). The undecidability of P (Q) in SR means that A (B) can neither prove nor refute P (Q) in SR. $P \Rightarrow Q$ is a theorem in B's (and not A's) frame; in other words, only B has the right to deduce Q in SR from an assertion of P made by A (if B happens to agree with A's assertion). Similarly, $Q \Rightarrow P$ is a theorem in A's (and not B's) frame. In fact $P \Rightarrow Q$ and $Q \Rightarrow P$ are illegitimate propositions in A's and B's frames respectively. The idea behind these restrictions is to allow A (B) to consider the truth of Q (P) without undermining the Lorentz transformations. A and B accept each other's observations/theorems as true/valid

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if and only if there is no disagreement with (or a refutation of) the observations or any step used in the proof of the said theorems.

In particular, Q is undecidable in SR, which means, as noted above, that B can neither prove nor refute Q in SR. The question we wish to consider is as follows. Given that A has asserted the truth of P, and given that $P \Rightarrow Q$ is a theorem of SR in B's frame, can B accept A's assertion and conclude Q? In the metatheorem that follows, we argue that B in fact has a formal refutation of A's assertion; i.e., B has a proof of $\neg P$ in SR and hence B has no way to conclude Q despite A's assertion of P. However, B does not have the right to use $Q \Rightarrow P$ along with the said proof of $\neg P$ to deduce $\neg Q$, because, as noted above, $Q \Rightarrow P$ is a theorem of A's (and not B's) frame. Hence Q continues to remain undecided in SR (in B's frame) despite A's assertion of P.

Metatheorem. *Suppose* A *claims the truth of* P*.* B *has a proof of* $\neg P$ *in* SR*. Formally,* B *must accept this proof rather than* A*'s claim. Hence the theoremhood of* $P \Rightarrow Q$ *does not decide* Q *in* SR *from* B*'s point of view. From the completeness theorem of* FOPL*, it follows that from* B*'s point of view, there must exist a model for* SR *in which* Q *is false despite* A*'s claim of the truth of* P*. The existence of such a model would make* SR *inconsistent from* A*'s point of view, because* A *would have to accept* B*'s derivation of* $\neg P$ *from* $\neg Q$ *as valid and also* B*'s assertion of* $\neg Q$ *as true. If such a model does not exist, then* SR *is inconsistent from* B*'s point of view.*

The proof (by contradiction) is as follows. Suppose B accepts A's claim of P as true; then B deduces Q from the theorem $P \Rightarrow Q$ of B's frame. From Q, B concludes $\neg P$. In other words, B refutes A's claim of having seen B's future at the instant of coincidence with A. From B's point of view, the (distant) event C has not yet happened at this instant and indeed, *need not happen at all*; B concludes that this assertion must hold globally for every observer, including A. It is important to note that the converse of the metatheorem does not hold; A cannot likewise refute a claim of Q by B because from A's point of view, such a claim could follow from a wrong definition of simultaneity in B's frame. It follows that every inertial observer deduces that SR is inconsistent. The Platonic assertion that SR is 'really' consistent despite this fact is unacceptable according to formalism. See [1] and [2] for further details, examples, and also for a criticism of non-Euclidean geometries. In [3], it is argued, using Gödel's second incompleteness theorem, that the proposition "A (B) is an inertial observer in $U(IBC)$" is unprovable in SR. Consequently, the completeness theorem of FOPL requires that there must exist a model for SR in which A (B) is a non-inertial observer in $U(IBC)$; this fact almost certainly makes SR logically inconsistent. To sum up, the incompleteness required by Gödel's theorems is not permitted in SR.

References

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