Physical and Geometrical Interpretations of the Riemann Tensor, Ricci Tensor, and Scalar Curvature

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Abstract

For a long time I have been very impressed with the various different levels at which Electromagnetism is taught. Beginning with the very simple electrostatics that my son learns in his grade, up to the elegant covariant formulations that serve as the basis for QED.

However, I am most impressed by the many wonderfully useful, clear, and intutive pictures that are used to explain Maxwell's Equations at a medium level: the level taught in lower and upper division physics courses, usually when introducing the integral forms of the equations. In this picture, Maxwell's equations could be explained in the following way. Charges are the sources of electric fields. Magnetic fields have no sources and must therefore run in continuous loops. Such vorices of magnetic field are created by currents. A changing magnetic field can cause a similar vortex of electric field. A changing electric field may act as a current to create a vortex of magnetic field. These, sentences contain the essence of Maxwell's equations and can be explained verbally and pictorially without complex mathematics.

By contrast, gravity is rarely taught at an intermediate level. The basics are introduced in early elementary school, Newton's law of gravitation is taught early in first year physics courses, and then the topic is largely forgotten until at least the second year of graduate study. I believe this is because we have not developed and/or popularized

adequate pictures to teach the essence of General Relativity, and how curved space results in gravitational forces, at an intermediate level.

In this paper I attempt to begin to rectify this situations, by describing various ways to interpret not only the Riemann Tensor, but also the Ricci Tensor, the scalar curvature, and Einstein's tensor (which is in fact the entity that couples to energy). The interpretations of the Riemann Tensor are standard in most graduate texts. I have taken inspiration from John Baez and Richard Feynman in developing and proving the interpretations of the Ricci Tensor, Einstein's Tensor, and Scalar Curvature.

The essence of General relativity can then be written in this way. The Riemann tensor governs the relative acceleration of two points moving in a curved space. The Ricci Tensor governs the changing size of a small volume propagating through a curved space. The scalar curvature represents the deviation of a volume's boundary, from that expected in a flat space. Einstein's tensor represents the scalar curvature of any three dimensional subspace. Einsteins equation implies that the scalar curvature of any three dimensional subspace must equal the energy density in that subspace.

From these basic concepts I derive Newtonian Gravity from Einstein's Gravity without appealing to complicated tensor manipulations. It is interesting that in this formulation we actually get the correct bending of light passing near a massive body. By contrast, straight Newtonian Gravity predicts this incorrectly by a factor of two.

The current paper focuses on presenting these concepts to those already familiar with Einstein's Gravity. Therefore, it relies on tensor manipulations and equations to justify and prove the physical interpretations of the various curvature tensors. I have also written a talk on the subject which removes most of these references and focuses instead on the results. Another version of the paper could be written which would focus on the results, with less reference to the proofs.