Estimating Canadian Monetary Policy Regimes*

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Abstract

Andolfatto and Gomme (2003) find evidence that Canadian monetary policy appears to alternate between high and low money growth rate regimes, and that private-sector belief formation over these unobserved regimes could induce significant persistence in the propagation of monetary policy shocks. In this paper, we examine the sensitivity of these conclusions by re-estimating the data allowing for the possibility of multiple regimes. In doing so, we find evidence of three (rather than two) distinct monetary policy regimes. In particular, we find that one policy regime is characterized by high money growth with moderate variability. The other two policy regimes are characterized by a common low money growth rate; they are distinguished primarily by their variability (high and low). A simulation exercise based on our three-regime model reveals an improvement in accounting for the behavior of the Canadian economy over some episodes; notably, the sharp increase in interest rates and the curtailment of economic activity in the early 1980s.

Keywords: monetary policy, regime switching, beliefs

JEL classification codes: E52, E42, E31, E13

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1 Introduction

In an earlier work, (Andolfatto and Gomme, 2003), we argued that monetary policy in Canada might be usefully described as a regime-switching process characterized by some degree of uncertainty on the part of agents over the prevailing policy regime in place at any given time. Faced with this uncertainty, agents are compelled to make inferences over the nature of monetary policy on the basis of observables, such as historical money growth rates and any other relevant information. We demonstrated that optimal learning behavior on the part of agents induces a sluggish adjustment in belief formation, so that expectations naturally tend to lag the underlying reality (as in Muth, 1961). We argued further that the endogenous propagation mechanism embedded in optimal learning over uncertain regimes has quantitatively important effects and can go some way in explaining the persistence in inflation expectations and interest rates following a change in monetary policy regime.

In this paper, we are concerned with investigating the sensitivity of our previous results to some restrictive assumptions made there in estimating policy regimes. In particular, our earlier paper restricts monetary policy to follow a two-state regime-switching process, with each regime characterized primarily by the underlying 'long-run' rate of money growth. Given the historical record on money growth rates in Canada, this assumption essentially 'forced' our estimation procedure to represent monetary policy as alternating between 'high' and 'low' (or 'loose' and 'tight') regimes. While this is perhaps not a bad approximation, it leaves open the question of whether this finding is primarily an artifact of restricting the estimation procedure in this way, and whether the quantitative predictions of our model might survive a more general specification.

Hamilton's (1989) original code only allowed for two regimes; it has since been extended to arbitrarily many regimes. We use this newer code to estimate two-, three- and four-regime processes. As in our earlier paper, we assume that monetary policy is described by a stochastic process for base-money growth. A regime is characterized by two parameters; one of which describes the underlying 'long-run' (persistent) money growth rate, and another that describes the variance of money growth (a transitory component). While our estimation allows for the possibility of many regimes, we find that empirically, the evidence supports the existence of only three regimes (suggesting that the limitation to two regime in our previous paper is restrictive). These three regimes are characterized as follows. Regime 1 exhibits high money growth with moderate variability; regime 2 exhibits low money growth with high variability; and regime 3 exhibits low money growth with low variability.

Here then, we find something new and interesting. In particular, since the average money growth rates in regimes 2 and 3 are fairly close, the analysis in this paper makes it clear that a regime is characterized not only by a change in average money growth, but also its volatility. Since the average money growth rates in regimes 2 and 3 are quite similar, the chief means by which agents would infer, say, a switch from regime 3 (low money growth, low volatility) to regime 2 (low money growth, high variability) is by observing a change in the variability of the money growth process. For agents in the model developed below, detecting this regime change is important since the likelihood of subsequently moving into regime 1 (high money growth, moderate variability) is much higher from regime 3 than from regime 2.

It is also interesting to note how the subtle distinction between the two low money growth regimes translates into inflation expectations. Suppose, for example, that individuals are confident of the regime that is in place. Then while the 'long-run' money growth rates in regimes 2 and 3 are virtually identical, inflation expectations are lower in regime 3. The reason for this is simple: the probability of transiting to the high money growth regime is low in regime 3 relative to regime 2.

Relative to our earlier findings, we find that the three-regime specification implies a much larger difference in long-run money growth rates across regimes. A transition from regime 1 to regime 2 now implies a fall in the average quarterly money growth rate of around 1.75 percentage points whereas in Andolfatto and Gomme (2003), the two regimes differed by 1.1 percentage points. Consequently, the size of the liquidity effect implied by our model is considerably larger than previously estimated. The three regime specification employed in the current paper is, then, better able to account for the behavior of the Canadian economy during Canada's disinflation episode in the early 1980s when interest rates rose sharply with the curtailment of real economic activity. The two-regime process employed in our earlier work could account for only some of the persistent effects of the Canadian disinflation of the early 1980s. The three-regime process does little better on this count since agents actually learn of a change in regime faster than in the two regime case. There are two reasons for this result. First, the difference in money growth rates in the three regime case is larger making it easier for agents to discern a change in regime. Second, the variance of the innovation to money growth in regime 2 is quite high making which also makes it easier for agents to infer when a regime change has occurred.

Our paper is organized as follows. Section 2 reports the results of our empirical investigation on Canadian base-money growth data. Here, we estimate the parameters of our generalized regime-switching process and use these estimates to interpret the data. In Section 3, we present a calibrated dynamic general equilibrium model that incorporates the estimated regime-switching process; this model is calibrated in Section 4. Section 5 then performs a number of impulse-response experiments designed to investigate how the model economy responds to various shocks. Section 6 provides a summary and conclusions.

2 Estimated Money Growth Process

Money growth follows an autoregressive process:

$$\mu_t - \overline{\mu}_t = \psi(\mu_{t-1} - \overline{\mu}_{t-1}) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2) \tag{1}$$

where μ_t is the money growth rate and $\overline{\mu}_t$ is the long run money growth rate. A money growth regime is characterized by a long run money growth rate, $\overline{\mu}_i$, and a standard deviation of the innovation ε_t, σ_i. Regimes evolve as a first-order Markov process:

$$\text{prob}[(\overline{\mu}_t, \sigma_t) = (\overline{\mu}_j, \sigma_j) \| (\overline{\mu}_{t-1}, \sigma_{t-1}) = (\overline{\mu}_i, \sigma_i)] = \pi_{ij}. \tag{2}$$

The parameters to be estimated are: the long run money growth rates, $\{\overline{\mu}_i\}$; the standard deviations of the innovations, $\{\sigma_i\}$; the autoregressive parameter, ψ; and the transition probabilities,

$\{\pi_{ij}\}$, where $\pi_{ij} \geq 0$ and $\sum_j \pi_{ij} = 1$. The parameters are estimated using a procedure related to that of Hamilton (1989).

Table 1 and Eqs. (3)–(5) summarize the estimates for 2, 3 and 4 regimes. The parameter estimates for 2 regimes are quite close to those reported in Andolfatto and Gomme (2003), with the differences attributable to the longer sample used in the current paper. The likelihood ratio test statistic for 3 versus 2 regimes is 25.8574. The 3 regime process has 6 more estimated parameters; the restriction that these 6 extra parameters are zero is easily rejected; for example, $\chi^2_{0.005}(6) = 18.5476$.[1] Next, consider 3 versus 4 regimes. In this case, there are 7 additional parameters and the likelihood ratio test statistic is 10.442. In this case, the restriction that the extra 7 parameters are zero cannot be rejected at conventional levels of significance; for example, $\chi^2_{0.1}(7) = 12.017$. On the basis of these results, it seems that 3 regimes best describes the data.

There are a number of interesting features of the estimated 3 regime process. To start, the estimated mean money growth of regimes 2 and 3 are very close to each other: 0.95% and 0.8% per quarter, respectively. What distinguishes the two regimes is the variability of money growth; the standard deviation of the innovation to money growth in regime 2 is more than $6\frac{1}{2}$ times larger than that in regime 3. The fact that the essential difference between regime 2 and 3 is the variability of money growth implies that in simulating regime changes in the general equilibrium model presented below, it will be essential to include the within-regime money growth shocks since these are vital to distinguishing between these two regimes.

Next, the transition matrix in Eq. (4) implies that transitions directly between regimes 1 and 3 are almost impossible since the estimated probabilities are very close to zero. A transition from regime 1 (high money growth, moderate variability) to regime 3 (low money growth, low volatility) almost always involves transiting through regime 2 (low money growth, high variability); likewise for transitions from regime 2 to regime 1. Further, regimes are fairly long lived. For example, regime 1 has a continuation probability of just over 0.97, implying an average duration for this

[1] Some of the estimated probabilities in Eqs. (4)–(5) are very close to zero and, with different code, could be restricted to equal zero. Such restrictions should not affect the degrees of freedom in the above likelihood ratio tests since the data has spoken as to the value of these probabilities (that they are essentially zero).

Table 1: Regime-switching estimates: Canadian per capita money base growth, 1955Q2-2006Q1

Parameter	2 Regimes	3 Regimes	4 Regimes
μ_1	0.015238	0.027036	−0.012942
	(0.002975)	(0.003902)	(0.002601)
μ_2	0.007733	0.009492	0.007406
	(0.001459)	(0.002891)	(0.001520)
μ_3		0.007934	0.023601
		(0.001373)	(0.003319)
μ_4			0.023601
			(0.001280)
ψ	0.440041	0.336354	0.589381
	(0.067141)	(0.073534)	(0.041787)
σ_1	0.000192	0.000056	0.000124
	(0.001014)	(0.001026)	(0.003124)
σ_2	0.000035	0.000228	0.000036
	(0.000603)	(0.001625)	(0.000461)
σ_3		0.000035	0.000066
		(0.000655)	(0.000818)
σ_4			0.000066
			(0.000436)
LLF	641.797996	655.446669	663.703788

Transition probabilities, 2 regimes:

$$\Pi = \begin{bmatrix} 0.960220 & 0.039780 \\ 0.048927 & 0.951073 \end{bmatrix} \tag{3}$$

Transition probabilities, 3 regimes:

$$\Pi = \begin{bmatrix} 0.970554 & 0.029446 & 3.44434 \times 10^{-12} \\ 4.99221 \times 10^{-6} & 0.909133 & 0.090862 \\ 0.011148 & 0.054886 & 0.933966 \end{bmatrix} \tag{4}$$

Transition probabilities, 4 regimes:

$$\Pi = \begin{bmatrix} 0.242928 & 0.404001 & 0.077158 & 0.275913 \\ 0.047666 & 0.889793 & 3.04189 \times 10^{-9} & 0.062541 \\ 9.01096 \times 10^{-13} & 0.022088 & 0.977908 & 4.30013 \times 10^{-6} \\ 0.321266 & 0.602429 & 1.16723 \times 10^{-8} & 0.076305 \end{bmatrix} \tag{5}$$

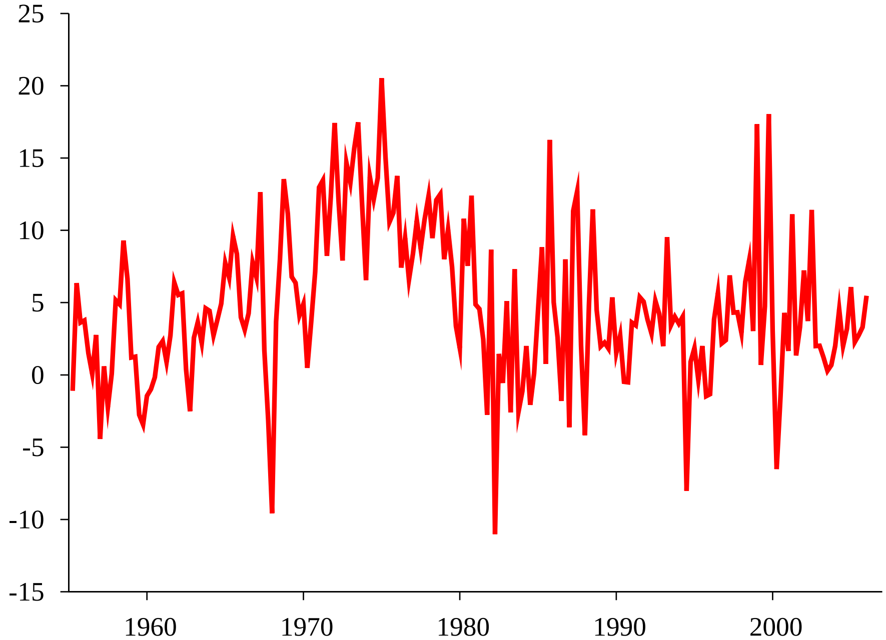


Figure 1: Canadian Base Money Growth

regime of 34 quarters. Regimes 2 and 3 are less persistent; their average durations are 11 quarters and 15 quarters, respectively. However, given the similarity in their means, it may be more interesting to consider the persistence of regimes 2 and 3 collectively; their joint average duration is 89.7 quarters.

Fig. 1 presents base money growth while Fig. 2 plots the regime probabilities for the 3 regime process. Following a "false start" in 1967, regime 1 (high money growth, moderate volatility) is the most likely regime from 1971 to mid-1981. This is the only period during which regime 1 is the most likely. Regime 3 (low money growth, low volatility) is associated with the period up to 1967, from mid-1988 to late-1998, and since early 2003. Regime 2 (low money growth, high volatility) is the highest probability regime from early 1967 to late 1968, late 1981 to early 1989 (the transition from high to low money growth), and from early 1999 to early 2003.

It is interesting to compare this reading of Canadian monetary policy with the one viewed through the lens of the two regime process. As shown in Fig. 3, the economy starts in the low money growth, low variance regime, then switches to the high money growth, high volatility regime in early 1967. With the exception of three quarters in 1970, the money growth process

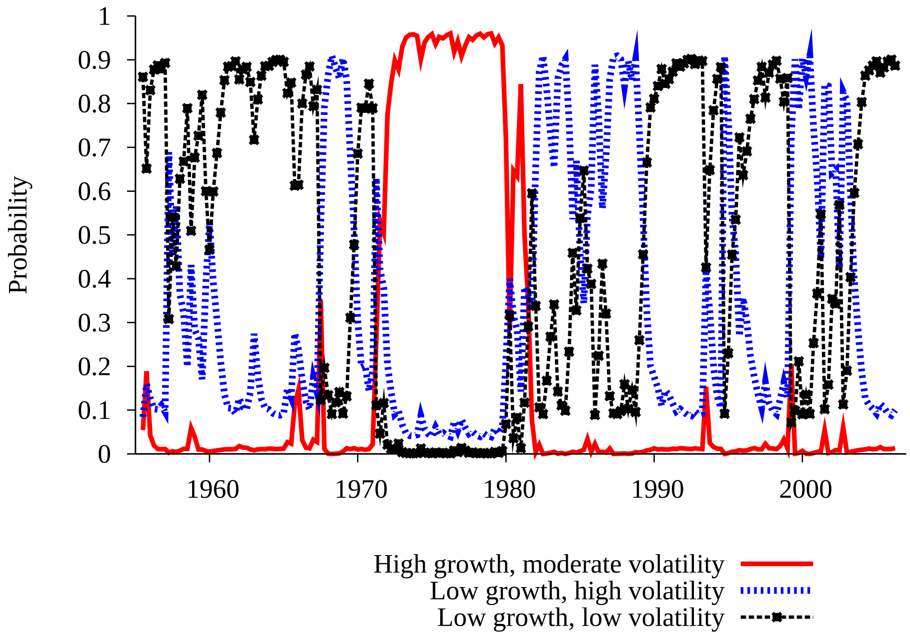


Figure 2: Regime Probabilities, Three Regime Process

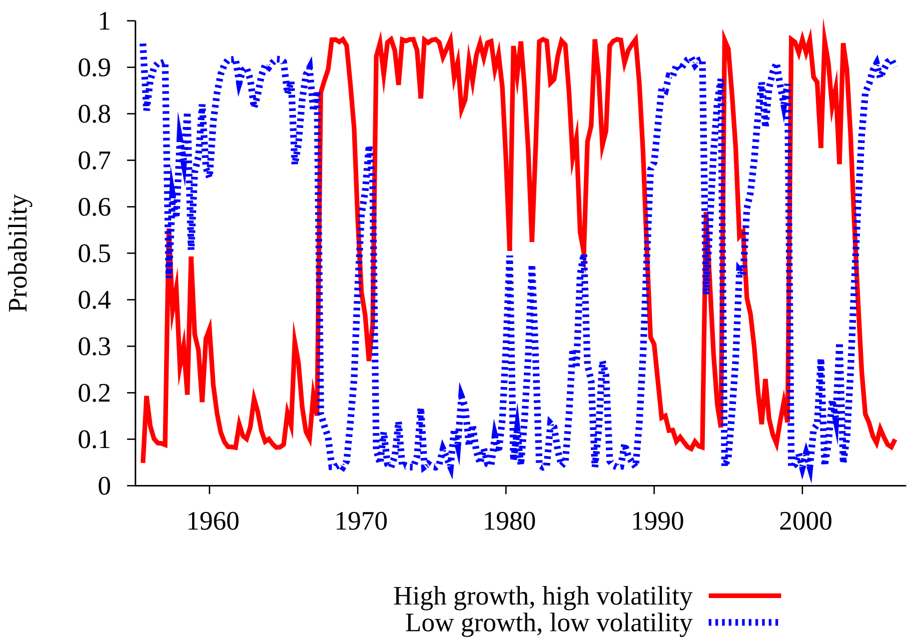


Figure 3: Regime Probabilities, Two Regime Process

does not switch back to the low money growth, low variance regime until late 1989. In late 1994, money growth again switches to high money growth, returning to low money growth in early 1996. The high money growth regime emerges once gain in early 1999, then disappears once more in late 2003. Allowing for three regimes allows the estimation procedure to distinguish between changes in average money growth on the one hand, and changes in its volatility on the other. Whereas the inference from the two regime process is that the money growth regime was high throughout the 1980s, the three regime process dates the switch out of the high money growth regime to the early 1980s. This dating corresponds more closely to narrative descriptions of Bank of Canada policy-making; see Howitt (1986). With two regimes, the volatility in money growth in the mid-1990s and in the late-1990s/early 2000s is seen as a switch back to the high money growth, high variance regime. By way of contrast, with three regimes these periods are interpreted as switches to regime 2 (low money growth, high volatility).

Finally, look at the Bank of Canada's inflation targeting regime through the lens of the regime switching estimates. The Bank of Canada formally adopted an inflation target in early 1991. This announcement had little immediate impact on the probabilities attached to regimes – regime 3 with low money growth and low variability was well-entrenched by that date, and had been for over a year-and-a-half. There is a considerable amount of turbulence in the probabilities attached to regimes 2 and 3 (both low money growth) from mid-1993 through to mid-1995. Interestingly, Bordo and Redish (2006) point to a move to greater transparency in Canadian monetary policy starting in 1994, in the midst of this period of shifting probabilities between the two low money growth regimes. More recently, from mid-1999 through mid-2003, regime 2 (low money growth, high volatility) typically has the highest probability. While the Bank of Canada's inflation targeting experience has been largely a success, this success has been difficult to discern from a traditional measure of the stance of monetary policy, the growth rate of the base money.

3 The Economic Environment

The model is essentially identical to that of Andolfatto and Gomme (2003). As such, apart from the money growth process, the model is quite similar to that of Fuerst (1992).

3.1 The Representative Household

The household has preferences over consumption, c_t, and leisure, ℓ_t, summarized by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t), \quad 0 < \beta < 1 \tag{6}$$

where the period utility function is well-behaved.

The household faces a cash-in-advance constraint on its purchases of consumption goods:

$$P_t c_t \leq M_t^c \tag{7}$$

where P_t is the price level and M_t^c is cash balanced brought into period t in order to finance purchases of consumption.

The household also faces a constraint on its time:

$$\ell_t + n_t \leq 1 \tag{8}$$

where n_t is the fraction of time spent working.

The household's budget constraint is

$$P_t(c_t + i_t) + M_{t+1}^c + M_{t+1}^d \leq W_t n_t + R_t k_t + M_t^c + (1 + R_t^d) M_t^d + \Pi_t^b + \Pi_t^g. \tag{9}$$

M_t^d represents funds on deposit at the bank; this allocation was chosen in the previous period, and earns the nominal return R_t^d. W_t is the nominal wage while R_t is the nominal rental rate on capital. Π_t^b and Π_t^g are profits remitted from banks and goods producing firms, respectively. k_t is capital brought into the period, and i_t is (real) investment. Capital evolves as

$$k_{t+1} = (1 - \delta) k_t + i_t. \tag{10}$$

3.2 Goods Producing Firms

The typical firm borrows at nominal interest rate R_t^b to finance its wage bill, $W_t\tilde{n}_t$ where $\tilde{n}_t$ is its choice of labor. The firm also rents capital, $\tilde{k}_t$. Output is produced according to a neoclassical production function, F. The firm's (static) problem is

$$\max \Pi_t^g = P_t F(\tilde{k}_t, \tilde{n}_t) - (1 + R_t^b) W_t \tilde{n}_t - R_t \tilde{k}_t. \tag{11}$$

3.3 Banks

At the start of a period, the typical bank has on deposit funds $\tilde{M}_t^d$ received from the household at the end of the previous period. The bank receives a nominal injection, X_t, from the monetary authority. These funds are lent to goods producing firms to finance their wage bill:

$$\tilde{M}_t^d + X_t \geq W_t \tilde{n}_t. \tag{12}$$

The bank's profits are

$$\Pi_t^b = (1 + R_t^b) W_t \tilde{n}_t - (1 + R_t^d) \tilde{M}_t^d. \tag{13}$$

Competition and free entry to the banking sector implies equalization of the interest rates on deposits and lending since intermediation is costless. The bank's profits are, then,

$$\Pi_t^b = (1 + R_t^d) X_t. \tag{14}$$

3.4 The Monetary Authority

Money growth occurs through injections to banks: $X_t = \mu_t M_t$ where μ_t is the net growth rate of money and M_t is total money: $M_t = M_t^c + M_t^d$. Total money balances evolve as

$$M_{t+1} = (1 + \mu_t) M_t. \tag{15}$$

The key aspects of money growth have been discussed in Section 2.

3.5 Information

Two informational assumptions are considered. First, suppose that the monetary policy regime, $(\overline{\mu}_i, \sigma_i)$, is known to the public. This case will be termed "full information." In this case, the only uncertainty that arises is with respect to future regimes (the Markov switching) and the money control error (ε_t).

Alternatively, suppose that current, past and future monetary policy regimes cannot be observed by the public. In this "incomplete information" case, private agents face not only the uncertainty associated with the full information scenario, but also with respect to the current regime. Let b_t^i denote the likelihood or probability that private agents attach to regime i being in place at time t having observed money growth rates up to and including date t. Let N be the number of regimes. The probabilities $\{b_t^i\}_{i=1}^N$ are updated using Bayes' rule:

$$b_{t+1}^j = \frac{\sum_{i=1}^N b_t^i \pi_{ij} \,\mathrm{prob}[\mu_{t+1}|i,j]}{\sum_{i=1}^N \sum_{i'=1}^N b_t^i \pi_{ii'} \,\mathrm{prob}[\mu_{t+1}|i,i']} \tag{16}$$

$\mathrm{prob}[\mu_{t+1}|i,j]$ is the probability of observing money growth μ_{t+1} if regime i is in place at date t and regime j is in place at date $t+1$. From Eq. (1), this probability is the probability of the money control error,

$$\varepsilon_{t,t+1}^{ij} = (\mu_{t+1} - \overline{\mu}_j) - \psi(\mu_t - \overline{\mu}^i) \tag{17}$$

given the sequence of regimes just described. Recall that the money control error is assumed to be Normally distributed. Since b_t^i is the probability agents assigned to regime i being in place at t and π_{ij} is the probability of transiting from regime i to regime j, the numerator of Eq. (16) is the unconditional probability of observing money growth μ_{t+1} under regime j. The denominator of Eq. (16) is the probability of observing money growth μ_{t+1} under *any* regime at $t+1$. (Follow the same logic as for the numerator with the additional step of adding across the N possible regimes.)

Table 2: Parameter Values

β	discount factor	0.995
ω	weight on consumption/leisure	0.37625
γ	coefficient of relative risk aversion	1.0
α	capital's share of income	0.3
δ	depreciation rate of capital	0.02

4 Calibration

The utility function is of the constant relative risk aversion variety:

$$U(c,\ell) = \begin{cases} \frac{[c^{\omega}\ell^{1-\omega}]^{1-\gamma}}{1-\gamma} & \text{if } 0 < \gamma < 1, \gamma > 1 \\ \omega \ln c + (1-\omega)\ln \ell & \text{if } \gamma = 1. \end{cases} \tag{18}$$

The goods production function is Cobb-Douglas:

$$F(k,n) = k^{\alpha} n^{1-\alpha}. \tag{19}$$

The parameters governing tastes and technology are set to fairly conventional values; see Table 2. These parameter values imply that agents work about $\frac{1}{3}$ of the time, and a real interest rate of 0.5% per quarter. The parameters governing monetary policy are as estimated in Section 2 for the three regime process.

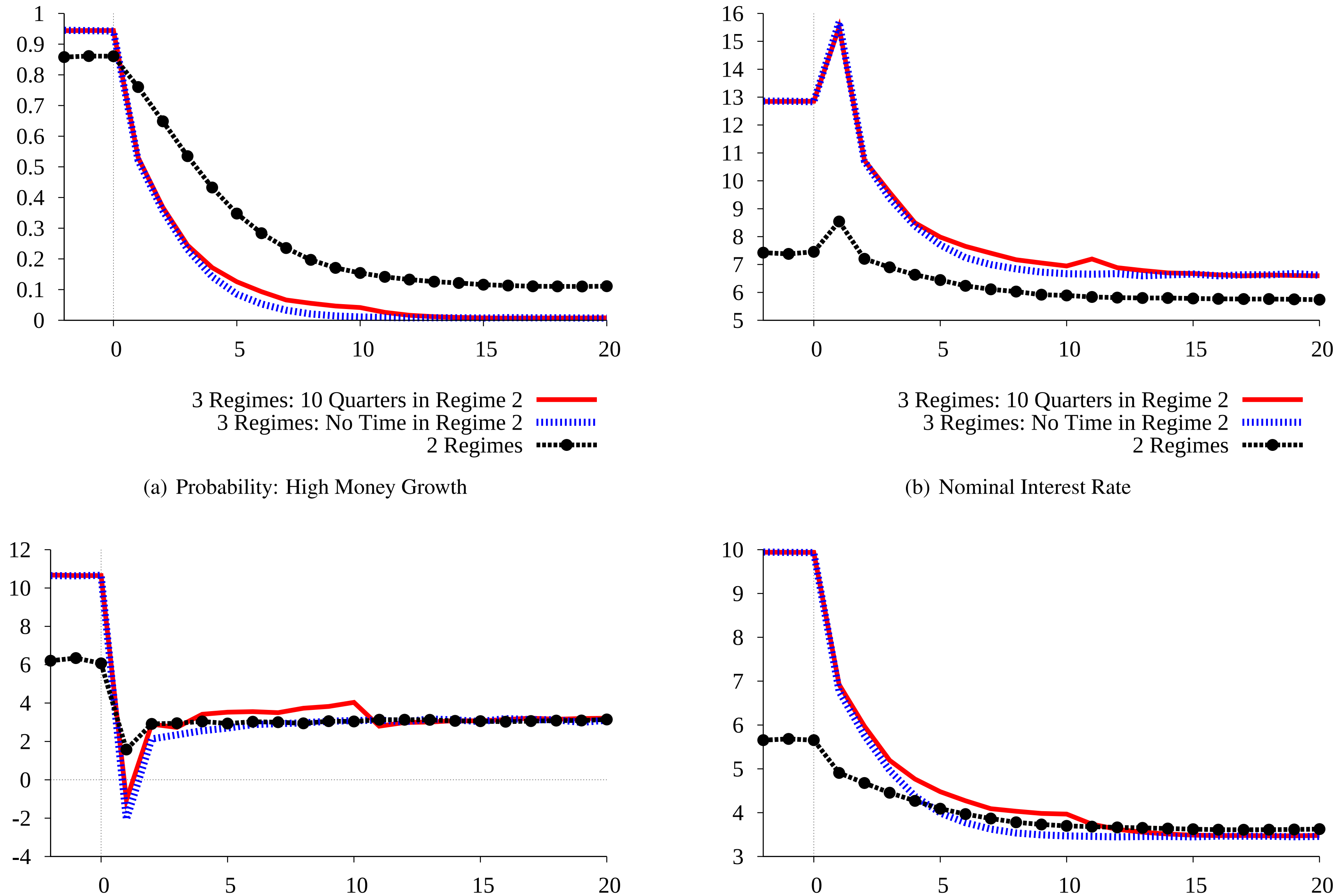
5 Simulating Regime Changes

As discussed in Section 2, in the three regime process, the principal distinction between regimes 2 and 3 is the variance of the innovation to money growth; the mean money growth rates across these two regimes are very close. The application of Bayes' rule as presented in Eq. (16) incorporates the variance of these innovations, although this fact may not be immediately apparent. Specifically, the differences in variances shows up in the probability of observing particular money growth rates under alternative regimes. To be more concrete, suppose that the average growth rates of money in regimes 2 and 3 were equal. Since the innovation variance is higher under regime 2, the probability distribution of innovations to money growth is flatter and more dispersed under regime 2. Suppose that agents observed a sequence of money growth rates that fluctuated a great deal around the average. The likelihood of observing such a pattern of money growth is higher under regime 2 than regime 3 since regime 2 has fatter tails to the distribution of innovations. Over time, agents will end up assigning a higher probability to regime 2 than regime 3. Alternatively, if agents observe a sequence of money growth rates that are fairly tightly clustered around the average, they will come to place a higher probability on regime 3 than regime 2 since the probability distribution function of innovations is more peaked around the average under regime 3. The reason for this protracted discussion is to motivate the fact that within-regime money growth volatility must be maintained when simulating regime changes in order to provide agents with useful information in distinguishing between the two low money growth regimes. In order to see through the resulting randomness, the figures below present average responses over 5000 regime changes in order to obtain a 'typical' path.

Throughout, attention is focused on simulating a disinflation experience; the effects of a set of regime changes resulting in a run-up in inflation are fairly similar. In all cases, the regime change occurs at date 1. For the two regime process, simulating a disinflation is fairly straightforward since there are but two regimes. For the three regime process, it makes sense to (eventually) transit into regime 3 (low money growth, low volatility) since the Markov transition matrix implies that regime 3 has a longer duration than regime 2 (low money growth, high variance). Two scenarios are run for the three regime process. Under the first, the economy moves from regime 1 to regime 2, staying there for 10 quarters (the average duration of that regime), then to regime 3. In the second scenario, money growth switches directly from regime 1 to regime 3. Since the estimated probability of such a regime change is quite low, this second scenario gives an extreme alternative to the first scenario.

The probability assigned to the high money growth regime is presented in Fig. 4(a). The three regime process with a direct transition from regime 1 to regime 3 displays the most rapid decline in this probability while the two regime process exhibits the greatest persistence. On the face of it, it may seem odd that the two regime process has the highest persistence in this probability. After all, the probability of exiting the high money growth regime is not that much higher for the three regime process. There are a couple of factors at work. First, the difference between the high and low money growth rates is much higher in the three regime case. In applying Bayes' rule, when agents consider the possibility that money growth continues to be generated by the high money growth regime implies drawing an innovation farther into the tail of the distribution under the three regime process. For this reason, one should expect to see the probability assigned to the high money growth regime fall faster in the three regime case. Second, in the three regime case, the large innovation variance of regime 2 means that in applying Bayes' rule, agents are more likely to infer that a large deviation from average money growth represents a switch out of regime 1, the high money growth regime. This factor also works to hasten the fall in the probability assigned to the high money growth regime in the three regime case. In fact, even when there is a direct transition from regime 1 to regime 3, agents end up placing a sizeable probability on regime 2 (around 25%; not shown).

The path of the nominal interest rate is depicted in Fig. 4(b). In limited participation models, unanticipated changes in money growth are immediately transmitted to the real economy through the loan market. A negative money growth shock depresses the supply of loanable funds; the nominal interest rate must rise to equilibrate this market. In the current environment, a negative money growth shock may occur owing to a negative innovation within a regime, or due to a switch from high to low money growth. Since the high money growth rate is considerably higher in the three regime case, so is inflation (see Fig. 4(c)) in the high money growth regime, and consequently so is the nominal interest rate. Since the fall in money growth (and inflation) is much larger for the three regime process, the rise in nominal interest rate at the time of the regime change is considerably larger in this case: 2.8 percentage points as compared to 1.1 percentage points in the two regime case. While there is a fairly protracted adjustment towards the new long run stationary state (owing in large part of the length of time it takes for the probability assigned to the high money growth regime to fall), in no case does the nominal interest rate remain above its previous



(a) Probability: High Money Growth

(b) Nominal Interest Rate

(c) Inflation

(d) Expected Inflation

Figure 4: Simulated Disinflation

long run value for more than one quarter. Limited participation models have both a liquidity effect (operating through the loan market) and an anticipated inflation effect (operating primarily through the household's labor-leisure decision). For the nominal interest rate to remain high means that the liquidity effect must dominate the anticipated inflation effect. At the time of the regime change, the liquidity effect clearly dominates since the regime change in unanticipated (meaning that there can be no anticipated inflation effect). In subsequent periods, the liquidity effect simply is not large enough to keep the nominal interest rate above its previous long run value.

The behavior of inflation is presented in Fig. 4(c). The transition from the high money growth stationary state to the low money growth stationary state is very rapid, and is basically over in the period following the regime change. Fig. 4(d) shows how next period's expected inflation evolves. The sluggish adjustment path of expected inflation follows largely from the sluggishness of the probability assigned to the high money growth regime. The differences in long run money growth rates across regimes 2 and 3 suggests that inflation should be 0.64 percentage points, yet the actual difference is more like 0.5 percentage points. The explanation for this difference highlights some of the subtle forces at work in this model. To understand these forces, consider the complete information version of the model in which case agents know exactly what regime they are in. Expected money growth in regimes 2 and 3 is necessarily higher than suggested by the long run money growth rates in these regimes. This discrepancy is larger for regime 3 since there is essentially no probability of moving directly from regime 2 to regime 1; such a transition will occur through regime 3. Consequently, in regime 3 inflation expectations are roughly 0.25 percentage points higher than long run money growth; this discrepancy is close to 0.1 percentage points in regime 2.

Fig. 5(a) gives the path of hours worked, expressed as a percentage deviation from the high money growth long run average. The rise in the nominal interest rate following a regime change increases the effective wage paid by firms since they must borrow to pay their wage bill. While it is possible that the real wage received by workers could actually fall more than enough to offset the effect of the nominal interest rate, this is not the case as seen in Fig. 5(b). As a result, hours of work fall. The path for output, shown in Fig. 5(c), is qualitatively similar to that of hours

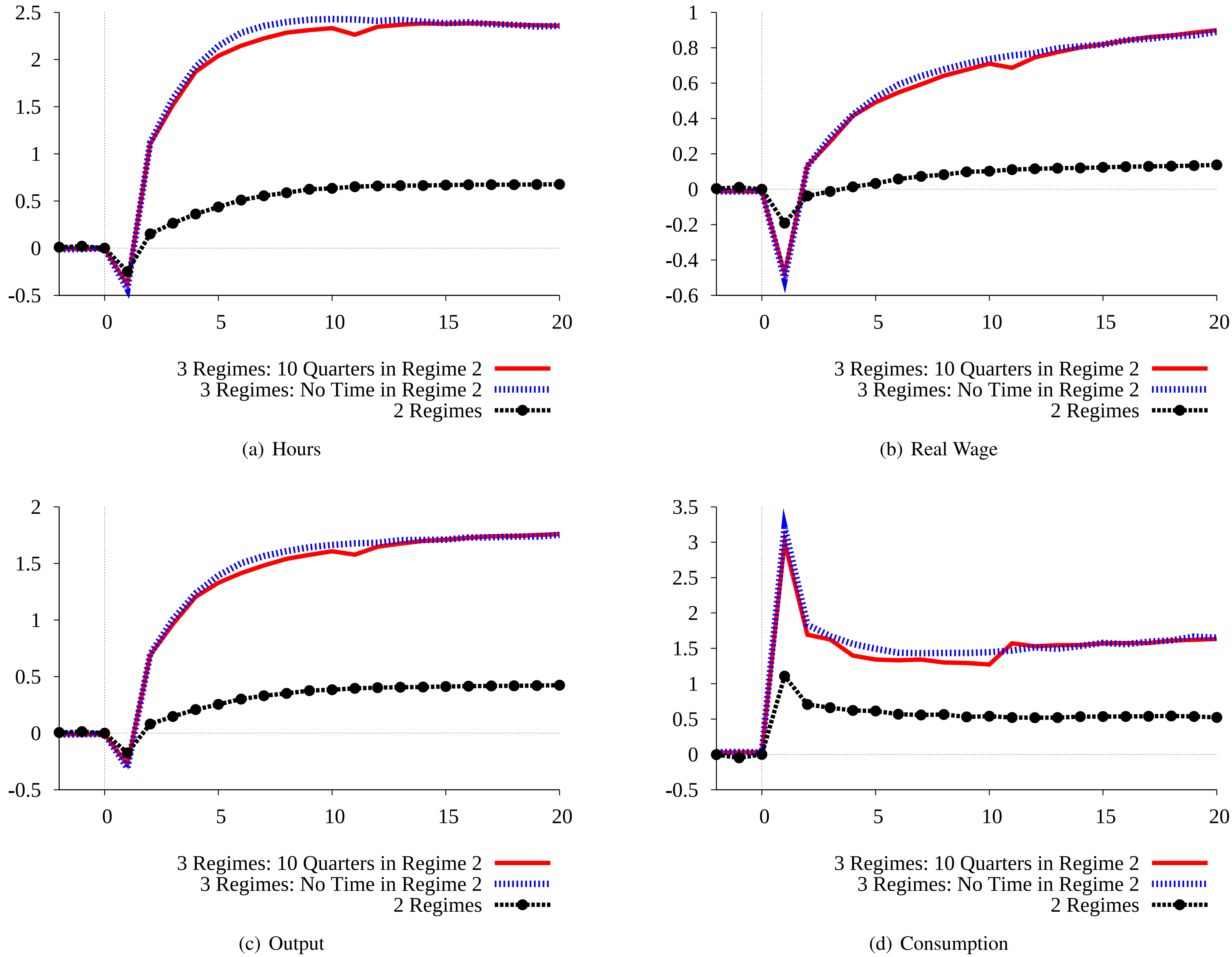


Figure 5: Simulated Disinflation

worked. Quantitatively, the impact effect on output and hours is quite similar across the two and three regime processes. In the longer term, output and hours rise more in the three regime case. To understand why this is so, recall that money growth in the high money growth regime is higher in the three regime case, while average money growth in the low growth regimes are all fairly similar. This higher money growth results in higher inflation and so a larger distortion in the household's labor-leisure choice. Consequently, when the economy is in the high money growth regime, output and hours will be lower in the three regime case. As a result, in the three regime case, the eventual rise in output and hours will be larger after the economy exits the high money growth regime.

The path of consumption is given in Fig. 5(d). The sharp increase in consumption at the time of the regime change reflects the effect of the unanticipated decline in inflation operating through the household's cash-in-advance constraint, Eq. (7). Given the fall in output shown in Fig. 5(c), this spike in consumption implies a sharp fall in investment (not shown). Over longer horizons, consumption remains above its previous stationary state value owing to the diminished distortions of the inflation tax.

6 Conclusions

In this paper, we extended our earlier two-regime analysis of Canadian monetary policy to allow for multiple policy regimes. Our empirical exercise reveals that our earlier restriction to two-regimes was modestly restrictive; our maximum likelihood estimates now reveal that Canadian base money growth is better described as a three-regime process. One regime is characterized by high money growth with moderate variability. The other two regimes are characterized by low money growth; but are distinguished by their variability (high and low). By and large, the estimated money growth process appears to fit the narratives describing recent Canadian experience.

Relative to our earlier work, the estimated impact of regime change on economic aggregates is now much larger on impact; but does not appear to affect the estimated adjustment process in any quantitatively significant manner. On the whole, sluggish belief formation appears to account for some–but not all–of the sluggishness typically observed in nominal variables.

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