1st International Workshop on Gerber-Shiu Functions

August 7-8, 2006

PROGRAM

August 6 (Sunday): EV 11.725

Registration/Cocktail	17:00-19:00	(Courtesy of the Dean, Faculty of Arts and Science, Concordia University)

August 7 (Monday): EV 2.260

Registration/Coffee	8:00-8:45	
Welcoming address	8:45-9:00	Chair: José Garrido
Talk - 1	9:00-10:00	**Elias S.W. Shiu**, University of Iowa "Ruin Theory by Divided Differences"
Talk – 2	10:00-11:00	**Gord E. Willmot**, University of Waterloo (Co-author: Jae-Kyung Woo) "On the class of Erlang mixtures with risk theoretic applications"
Coffee break	11:00-11:30	
		Chair: Qihe Tang
Talk – 3	11:30-12:30	**Hansjoerg Albrecher**, Austrian Academy of Sciences "Ruin theory in the presence of dividend payments and dependent risks"
Lunch	12:30-14:00	TIMS Back Room
		Chair: Elias Shiu
Talk – 4	14:00-15:00	**X. Sheldon Lin**, U. of Toronto (Co-author: Kristina Sendova, U. of Western Ontario) "Analysis of the Compound Poisson Risk Model with Multiple Thresholds: A Differential Equation Approach"
Talk – 5	15:00-16:00	**Etienne Marceau**, U. Laval (Co-authors: Helene Cossette, U. Laval and David Landriault, U. of Waterloo) "Analysis of ruin measures in renewal risk models with time dependent claim amounts"
Coffee break	16:00-16:30	
		Chair: Hansjoerg Albrecher
Talk – 6	16:30-17:30	**Jun Cai**, University of Waterloo "On the Time Value of Absolute Ruin with Debit Interest"

August 8 (Tuesday): EV 2.260

Coffee	8:30-9:00	
		Chair: Gordon E. Willmot
Talk - 1	9:00-10:00	**Qihe Tang**, University of Iowa "The Overshoot of a Random Walk with Negative Drift"
Talk – 2	10:00-11:00	**Andrei Badescu**, U. of Toronto (Co-author: Soohan Ahn, University of Seoul) "On the Analysis of the Gerber-Shiu Discounted Penalty Function for Risk Processes with Markovian Arrivals"
Coffee break	11:00-11:30	
		Chair: Xiaowen Zhou
Talk – 3	11:30-12:30	**Steve Drekic**, U. of Waterloo (Co-authors: David Dickson, U. of Melbourne, Andrei Badescu, U. of Toronto and David Landriault, U. of Waterloo) "Analysis of a Dividend Barrier Strategy for a Class of Markovian Risk Models"
Lunch	12:30-14:00	TIMS Back Room
		Chair: X. Sheldon Lin
Talk – 4	14:00-15:00	**Cary Tsai**, Simon Fraser U. (Co-author: Yi Lu, Simon Fraser U.) "The Markov-modulated continuous time surplus process perturbed by diffusion"
Talk – 5	15:00-16:00	**David Landriault**, U. of Waterloo (Co-author: Gord Willmot, U. of Waterloo) "On the discounted penalty function in the renewal risk model with Coxian-distributed claim sizes"
Closing remarks	16:00-16:15	

August 9 (Wednesday):

Sightseeing Tour	15:00-18:30	Optional, First Class city bus tour

ABSTRACTS

August 7 (Monday):

Talk - 1 Elias S.W. Shiu, University of Iowa

"Ruin Theory by Divided Differences"

It used to be important for an actuary to know how to interpolate and extrapolate values from a numerical table. Thus, the calculus of finite differences was an integral part of an actuarial education; it was one-half of the old Part 3 examination of the Society of Actuaries until the early 1970's. Ruin theory was incorporated in the Part 5 examination of the Society of Actuaries about thirty years ago, but it was completely removed from the actuarial syllabus since 2000. This talk will introduce the concept of divided differences and show that some results in ruin theory can be simplified and unified by means of divided differences.

Talk – 2 Gord E. Willmot, University of Waterloo (Co-author: Jae-Kyung Woo)

"On the class of Erlang mixtures with risk theoretic applications"

The class of countable mixtures of Erlang distributions with the same scale parameter is a very tractable class of models. It is dense in the class of continuous positive random variables. In addition, it includes as members many distributions which are of much interest in various risk theoretic contexts.

In this talk we shall demonstrate that distributions such as mixtures and combinations of exponentials, generalized Erlangian distributions, and finite mixtures of Erlangs with different scale parameters are all members of the class, with mixing weights which are easy to calculate, often with the help of Panjer-type recursions.

We then discuss applications of these results in connection with aggregate claims and stop loss analysis, as well as in connection with ruin and Gerber-Shiu analysis. In particular, both finite and infinite time ruin probabilities in the classical Poisson risk model with mixed Erlang claim amounts are straightforward to obtain numerically. Also, applications involving the deficit at ruin, the density of the time of ruin, and compound geometric 'ladder height' representations are also considered.

Talk – 3 Hansjoerg Albrecher, Austrian Academy of Sciences

"Ruin theory in the presence of dividend payments and dependent risks"

Over the last years, ruin-related quantities of the collective risk process have been studied in the literature for various generalizations of the classical Cramer-Lundberg model. In this talk, some recent developments with respect to considering dependence among the involved risks and including dividend payments to shareholders will be discussed. For the latter extension, some optimality issues will be considered.

Talk – 4 X. Sheldon Lin, U. of Toronto (Co-author: Kristina Sendova, U. of Western Ontario)

"Analysis of the Compound Poisson Risk Model with Multiple Thresholds: A Differential Equation Approach"

The expected discounted penalty function proposed by Gerber and Shiu (1998) enables us to analyze the time of ultimate ruin and its related quantities associated with a renewal risk model in a unified manner. Using the renewal property of the model, an integro-differential

equation can be derived for the expected discounted penalty function. Hence methods in differential equations may apply. One of the advantages of this approach is that it requires little probabilistic argument, as opposed to a probabilistic approach, and thus can easily be understood by non-probabilists. Another advantage is that it can some time handle more complex risk models, especially the risk models with dividend strategies, for which probabilistic reasoning might be difficult.

In this talk, I will discuss how to use the differential equation approach to solve for the expected discounted penalty function for the compound Poisson model with multiple thresholds. I first derive a piece-wise integro-differential equation for the expected discounted penalty function and then provide a recursive scheme to obtain general solutions to the integro-differential equation. As a particular solution of the equation, the expected discounted penalty function is solved. As an illustration, the probability of ruin for the model is explicitly derived. Other applications of the general solutions may include the expectation of the present value of all dividends when the thresholds are interpreted as dividend barriers.

If time permits, I will discuss a recent joint work with Hans Gerber and Hailiang Yang on a dividends-penalty identity.

Talk – 5 Etienne Marceau, U. Laval (Co-authors: Helene Cossette, U. Laval and David Landriault, U. of Waterloo)
"Analysis of ruin measures in renewal risk models with time dependent claim amounts"

In this talk, we consider extensions to the compound Poisson risk model (Cramer-Lundberg) and the renewal risk model (Sparre-Andersen). Within the two original risk models, it is assumed that the interclaim times are independent of the amount of a claim. This hypothesis of independence may be inappropriate in some contexts to describe the mechanism underlying the arrival and the amount of claims. In our proposed extensions, we assume that the distribution of the jth claim amount X_j is function of the time T_j elapsed between the $(j-1)$th claim and the jth claim. Within these models, we examine the computation of ruin measures with the analysis of the expected value of the discounted penalty function. We find the Laplace transform and we derive the defective renewal equation for the expected value of the discounted penalty function. Numerical examples with different choices of distributions for X_j and T_j as well as different dependence structures are presented.

Talk – 6 Jun Cai, University of Waterloo
"On the Time Value of Absolute Ruin with Debit Interest"

Assume that the surplus of an insurer follows a compound Poisson surplus process. When the surplus is below zero or the insurer is on deficit, the insurer has to borrow money at a debit interest force to pay claims. Meanwhile, the insurer will repay the debts from her premium incomes. The negative surplus may return to positive level if the debts are at a safe level. However, when the negative surplus is below a critical level, the surplus is no longer able to return to positive level since the debts at this time would exceed the total premium incomes available after this time. Absolute ruin is said to occur at this moment, which is called the time of absolute ruin. Unlike the classical ruin problems, many questions about absolute ruin have not been solved.

In this talk, we consider absolute ruin problems by defining the expected discounted penalty function or the Gerber-Shiu function at absolute ruin. The function includes the absolute ruin probability, the surplus just before absolute ruin, the deficit at absolute ruin and many other quantities related to absolute ruin. First, we derive a system of integro differential equations satisfied by the Gerber-Shui function and obtain a defective renewal equation that links the integro-differential equations in the system. Then, we show that when the initial surplus goes to infinity, the absolute ruin probability and the classical ruin probability are asymptotically equal for large claims while the ratio of the absolute ruin probability to the classical ruin probability goes to a positive constant, which is less than one, for small claims. Finally, we derive explicit solutions for the Gerber-Shui function when claim sizes are exponentially distributed.

August 8 (Tuesday):

Talk – 1 Qihe Tang, University of Iowa

"The Overshoot of a Random Walk with Negative Drift"

Let $\{S_n, n \geq 0\}$ be a random walk starting from 0 and drifting to $-\infty$, and let $\tau(x)$ be the first time when the random walk crosses a given level $x \geq 0$. Consider as a special kind of Gerber-Shiu function the tail probability of the overshoot $S_{\tau(x)}-x$, associated with the event $(\tau(x) <\infty)$. In this talk we focus on the asymptotic behavior of this tail probability for the cases of heavy-tailed and light-tailed increments. The results enhance the classical Veraverbeke's (1977, Stochastic Processes and their Applications) formula.

Talk – 2 Andrei Badescu, U. of Toronto (Co-author: Soohan Ahn, University of Seoul)

"On the Analysis of the Gerber-Shiu Discounted Penalty Function for Risk Processes with Markovian Arrivals"

In this paper, we consider an insurance risk model governed by a Markovian arrival claim process and by phase-type distributed claim amounts, which also allows for claim sizes to be correlated with the inter-claim times. A defective renewal equation of matrix form is derived for the Gerber-Shiu discounted penalty function and solved using matrix analytic methods. The use of the busy period distribution for the canonical fluid flow model is a key factor in our analysis, allowing us to obtain an explicit form of the Gerber-Shiu discounted penalty function avoiding thus the use of Lundberg's fundamental equation roots. As a special case, we derive the triple Laplace transform of the time to ruin, surplus prior to ruin, and deficit at ruin in explicit form, further obtaining the discounted joint and marginal moments of the surplus prior to ruin and the deficit at ruin.

Talk – 3 Steve Drekic, U. of Waterloo (Co-authors: David Dickson, U. of Melbourne, Andrei Badescu, U. of Toronto and David Landriault, U. of Waterloo)

"Analysis of a Dividend Barrier Strategy for a Class of Markovian Risk Models"

We consider a class of Markovian risk models in which an insurer's surplus process is modified by the payment of dividends when the surplus level exceeds a certain threshold. We employ a probabilistic argument in conjunction with fluid flow methodology to obtain a general expression for the expected present value of total dividend payments made prior to ruin. Several special cases are discussed including the classical compound Poisson model, the Sparre Andersen model with phase-type inter-claim times, and risk models with correlated inter-claim times.

Talk – 4 Cary Tsai, Simon Fraser U. (Co-author: Yi Lu, Simon Fraser U.)
"The Markov-modulated continuous time surplus process perturbed by diffusion"

In this paper, we consider a Markov-modulated continuous time surplus process perturbed by diffusion. The types of claims, the rates of premiums, and the variances of the Weiner processes are influenced by an external Markovian environment process. Given the initial surplus and environment state, we derive a system of integro-differential equations for the conditional expected discounted Gerber-Shiu penalty functions at ruin due to a claim and oscillation respectively. We also propose a generalized Lundberg's equation and explicit expressions for the expected discounted penalty functions at ruin based on the two-state model. Finally, we give some applications from the derived conditional expected discounted functions.

Talk – 5 David Landriault, U. of Waterloo (Co-author: Gord Willmot, U. of Waterloo)
"On the discounted penalty function in the renewal risk model with Coxian-distributed claim sizes"

In the actuarial literature, the analysis of the Gerber-Shiu discounted penalty function in he Sparre Andersen risk model is often performed subject to the interarrival time distribution being in a class of distributions (see e.g. Dickson and Hipp (2001), Gerber and Shiu (1998, 2005), Li and Garrido (2004, 2005)). In this talk, we perform the general Gerber- Shiu type analysis of the renewal risk model with an arbitrary interclaim time distribution and Kr-distributed claim sizes. Contrary to the former class of renewal risk models, the general expression for the Gerber-Shiu discounted penalty function in the latter class (of renewal risk models) is obtained via a two-step process. Using a probabilistic approach, we first identify the form of its solution. Then, by conditioning on the time and the amount of the first claim and substitute in the form of the solution found in step 1, a fairly large subset of Gerber-Shiu functions can actually be evaluated. A few numerical examples are carried out to illustrate the procedure as well as to show the impact of the choice of the interclaim time distribution on various ruin related quantities.