GUIDELINES ON WRITING PROOFS

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What is NOT a proof?

1. Providing examples in which the statement holds is not a proof.
   Unless you can enumerate every possible case, it does not suffice as a proof just providing examples. However, you only need one counterexample to disprove a statement.
   For example, you are asked to prove that the sum of an even number and an odd number is odd. If you want to prove it by examples, you would have to check it for all even and odd numbers, which is an impossible task. However if you want to disprove the statement that the sum of an even number and an odd number is even, it suffices to give an example in which this does not hold.

2. Showing that both the condition and the result are true is not a proof.
   When proving a statement, we should always start with supposing the condition given is true, and use deduction to obtain the desired result. In almost all cases, the condition given is not always true, in which case it is futile to try to prove the given condition.

3. Saying the main point is obvious is not a proof.
   There are main points you need to attack in a proof, depending on the context in which the question is asked. Reserve the word "obvious" for those statements that follow directly from trivial applications of definitions or straightforward algebra.

How to write good proofs?

1. Precede your statement with "we know from X that…" if it follows immediately from X, where X could be a definition or a result that has been covered in the class or textbook(s). When you do your homework problems, I do not encourage you to use results in the textbook(s) yet not covered in the class. In many cases, if you invoke an advanced result the statement to be proved becomes trivial.

2. Use the format "Claim: …" before your statement when you are about to devote the following paragraph to proving this statement.

3. Number your equations or intermediate statements when the proof is long. Make explicit references to previously proven results, for example, it is preferable to write "by equation (2)" rather than "as shown above".

4. Avoid stating irrelevant facts. These include but are not limited to: restatement of basic definitions (unless it helps to clarify the goal of the proof), implications of the given condition that play no role in obtaining the desired result, excessive details, etc.

5. Put your statements in logical order. It is a good practice to use English words to connect parts of your proof and show your reasoning process.

6. Use good notation. Follow standard notation. Show all the arguments of a function whenever confusion is possible. Learn the precise way of making a statement rather than using vague phrases like "large enough", "close to zero", "almost equal", etc.

7. Be careful with inequalities. Distinguish weak inequalities from strict ones. When you have a sequence of inequalities and equalities, make sure each of them follows from definitions or known results. Clarify if necessary. You should not switch terms on both sides of an equality, when that breaks off the logical order. For example: $A = B$ follows from definition 1, $B \geq C$ follows from theorem 2, and your goal is to prove $A \geq C$. Then you should not write $B = A \geq C$. A good way of writing it would be:

$$\begin{aligned} A &= B \quad \text{(by definition 1)} \\ &\geq C \quad \text{(by theorem 2)} \end{aligned}.$$