

Econ 425/525 Homework 7

Department of Economics
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For practice only—no need to submit your answers.

Provide complete work that leads to each answer. Do the problems in the order given.

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1. Solve the maximization problem

$$\max_{x,y,z} \frac{x}{2} - y$$

subject to

$$\begin{aligned} e^{-x} + az^2 &\leq y + b, \\ x &\geq 0, \end{aligned}$$

where a and b are positive constants.

2. Sundaram, #3, p. 198.
3. (HMC 1.3.20.) Suppose \mathcal{D} is a nonempty collection of subsets of \mathcal{C} . Consider the collection of events,

$$\mathcal{B} = \cap\{\mathcal{E} : \mathcal{D} \subset \mathcal{E} \text{ and } \mathcal{E} \text{ is a } \sigma\text{-algebra}\}.$$

Note that $\phi \in \mathcal{B}$ because it is in each σ -algebra, and, hence, in particular, it is in each σ -algebra $\mathcal{E} \supset \mathcal{D}$. Continue in this fashion and show that \mathcal{B} is a σ -algebra.

4. Suppose P is the probability set function defined on \mathcal{B} , a collection of events based on sample space \mathcal{C} . Recall the three properties that it must satisfy. Prove the following properties of P .
 - (a) $P(\emptyset) = 0$.
 - (b) $P(C) = 1 - P(C^c)$.
5. (HMC 1.4.11.) If C_1 and C_2 are independent events, namely, $P(C_1 \cap C_2) = P(C_1)P(C_2)$, show that the following pairs of events are also independent: C_1^c and C_2 , C_1 and C_2^c , and C_1^c and C_2^c .
6. Devah Pager, a sociologist at Northwestern University, had students with fictitious resumes apply for work with 350 employers, for mainly low-wage, entry-level jobs. Some of the findings of the studies are: white applicants with no criminal records are called back for interviews 34% of the time, while white applicants with a criminal record 17% of the time; black applicants with no criminal records are called back for interviews 14% of the time, while black applicants with a criminal record only 5% of the time. (The following is made up for the purpose of this problem.) Let us suppose white applicants account for 80% of the applicants and black ones 20%. Further, assume that for both white and black applicants, 20% have criminal records. Calculate the following conditional probabilities.
 - (a) Conditional on an interview being given, what is the probability that the applicant is black? (In other words, what proportion of interviews is given to black applicants?)
 - (b) Given that an applicant is called back for an interview by a company and the applicant has a criminal record, what is the probability that the applicant is black?