

Econ 301 Homework 5: for Practice Only

Winter 2012

Provide complete work that leads to each answer. Do the problems in the order given.

Instructor: Ming Li

1. TRUE or FALSE (3 points each). All credits will be assigned to explanations.
 - (a) A firm using the production function $f(L, K) = 5L + 10K$ will experience diminishing marginal product of labor (L).
 - (b) Monotonic transformations of the same production function represent the same technology.
 - (c) A firm's long-run profit function is always at least as high as its short-run profit function.
 - (d) It is possible for the marginal cost curve to be always *strictly* above the average cost curve.
2. (10 points) A firm uses the technology $y = 2x_1^{1/2}x_2^{1/2}$ to produce a product. Let the factor prices be $w_1 = 4$ and $w_2 = 1$, and let output price be $p = 1$.
 - (a) (4 points) Does the technology exhibit constant, decreasing, or increasing returns to scale? Diminishing marginal product? Diminishing technical rate of substitution?
 - (b) (3 points) Suppose in the short run, factor 2 is fixed at $\bar{x}_2 = 4$. Determine the amount of x_1 used to maximize profits.
 - (c) (3 points) Now, suppose the price of factor 2 changes to $w'_2 = 2$, how does your answer to (b) change? What if the new output price is $p' = 3$?
3. (12 points) Consider the same technology $y = 2x_1^{1/2}x_2^{1/2}$ as that in the previous question. Let the factor prices be $w_1 = 4$ and $w_2 = 1$.
 - (a) (3 points) Suppose in the short run, factor 2 is fixed at $\bar{x}_2 = 4$. What are the minimum costs for producing 8 units of output?
 - (b) (3 points) In the long run, what are the long-run costs for producing 8 units of output?

- (c) (3 points) For any level of output y , what are your answers to parts (a) and (b)?
4. (12 points) Consider the cost function $c(y) = \frac{y^3}{2} + 1$.
- (4 points) Calculate the average cost, average variable cost, average fixed cost, and marginal cost functions.
 - (4 points) Find the lowest points of the average variable cost and average cost curves.
 - (4 points) Draw all three cost curves on the same graph.
5. (12 points) An ancient Asian king wants his palace painted. There are two colors from which to choose: red and yellow. Either color is fine for him as long as the walls are painted. There are 999 chambers in his palace. It takes one bucket of paint to paint each chamber.
- (3 points) Write down the production function that represents the production technology. The output is painted chambers, and the input factors are red paint and yellow paint. Draw the isoquant that corresponds to all chambers in the palace being painted.
 - (4 points) Suppose red paint costs 1 gold coin per bucket, and yellow paint costs 1 silver coin per bucket. It is known in this kingdom gold coins are twice as valuable as silver coins. To minimize the painting costs, how many chambers are painted in red, and how many are painted in yellow? What are the total costs?
 - (3 points) Suppose royal regulations specify that at most 500 buckets of yellow paint can be used each month and red paint on the other hand is not restricted. The king wants his palace painted within a month, in time for the new year. What are the minimum costs now?
 - (2 points) Note that the total cost has changed from part b to part c. Fixing the price of red paint at 1 gold coin per bucket, what condition must the price of yellow paint satisfy in order for the total cost not to be impacted by the regulations?