Information Collection in Bargaining*

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Abstract

I analyze a bilateral bargaining model with one-sided uncertainty about time preferences. The uninformed player has the option of halting the bargaining process to obtain additional information, when it is his turn to offer. In general, the uninformed player does not collect information when he is quite sure about his opponent's type. I identify preference settings in which the uninformed player collects information until he is sufficiently sure about his opponent's type, as long as the information source is accurate enough. The uninformed player is more likely to draw signals and is better off, if the information source is more accurate.

Key words: bargaining, alternate offers, incomplete information, delay.
JEL codes: C78, D82.

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1 Introduction

Incomplete information is widely viewed as an important reason for delays in bargaining. This paper aims to investigate a previously unstudied source of delay in bargaining when there is incomplete information. That is, the uninformed player may not want to make offers at all, unless he has sufficient information about his opponent.[1] He may instead choose to search for additional information. This consideration is especially important when the acquirement of a small amount of additional information may significantly improve the outcome for the uninformed player.

Consider a bilateral bargaining situation in which an informed player bargains with an uninformed player. The informed player could be either "strong" or "weak." The uninformed player prefers to have a "weak" opponent. The uninformed player may only want to start bargaining when he is sufficiently sure that his opponent is weak or strong.

To motivate the exercise in this paper, consider a district attorney prosecuting a case against a crime suspect. The defense has private information about the merits of the case. Both sides prefer to arrive at a deal earlier and avoid going to trial. However, if the district attorney chooses to wait, he may come upon clues that reveal information about the defendant. Therefore, he may not want to offer or accept a deal until he has sufficient information about the case.

Another example is negotiations between two conflicting countries. The leader of one of them may face a political crisis at home and want to tout the resolution of the conflict as her achievement to diffuse the crisis. If so, she would be less patient. However, the leader of the other country does not know whether she faces a crisis or not. He may choose to collect more information before making offers.

Ausubel, Cramton, and Deneckere (2002) provide a comprehensive survey of the vast literature on bargaining with incomplete information. In one branch of the literature, the uncertainty lies in the informed player's value of the good being bargained upon. The time length between offers is exogenously given. The uninformed player uses increasingly generous offers to screen the informed player. A strong informed player waits longer to arrive at agreements with the uninformed player. This phenomenon is sometimes referred to as the "Coasian Dynamics." The works by Sobel and Takahashi (1983), Fudenberg, Levine, and Tirole (1985), Grossman and Perry (1986), Gul and Sonnenschein (1988), and Gul, Sonnenschein, and Wilson (1986) are a few examples. Note that delays *can* happen in these games if arbitrary depen-

[1] In this paper, the uninformed player is referred to as "he", and the informed player "she."

dence on history of play is allowed. Ausubel and Deneckere (1989) show that, when there is no gap between the valuations of the uninformed and informed players, a "folk theorem" is true in the context of durable goods monopoly (which is technically equivalent to bilateral bargaining with one-sided incomplete information), in that the uninformed party's equilibrium payoff ranges from that when he has full monopoly power and that when he has no such power. Another smaller branch of literature allows the informed player to endogenously choose the amount of delay. In the selected equilibrium, stronger types incur longer delays (and refuse to return to bargaining), so as to capture a bigger proportion of the surplus. This is usually called "strategic delay." Admati and Perry (1987) initiate this line of research and Cramton (1992) extends it to the case of two-sided uncertainty.

In this paper, I adopt Rubinstein's (1985) model, in which the private information is about time preferences. In addition, I introduce an outside information source that the uninformed party can use if he halts the bargaining process. I assume that he gets a signal from this information source for each period he stays away from the bargaining table. I call this information collection.

I focus my analysis on pure strategy sequential equilibrium. Alternate-offer bargaining games of incomplete information have a plethora of sequential equilibria. The same is also true in my model. I adopt a version of refinement used by Rubinstein (1985) and Osborne and Rubinstein (1990), called bargaining sequential equilibrium or rationalizing sequential equilibrium. This refinement ensures that for the two-type model without information collection, for any distribution over the two types, there is a unique equilibrium satisfying the refinements.[2] Thus, the uninformed player is able to make a comparison between waiting and offering, since he knows what his payoff would be if he starts the bargaining process. This proves very important for the analysis in this paper.

The results of this paper are fairly intuitive. The uninformed player does not collect information, if his belief about the informed player being the weak type is very high or very low. I identify preference settings in which the uninformed player collects information until he is sufficiently sure about his opponent's type. I also identify preferences for which it is true that the uninformed player prefers to have a more accurate information source, and is more willing to collect information when the information source is more accurate.

I view information collection as a potentially important explanation of bargaining

[2] There could still be multiplicity at one or two threshold values. In those cases, the equilibrium with shorter delay is selected.

delays in many situations that complements those explanations mentioned above, since it works through a very different mechanism.

2 The Model

The model is adapted from the standard alternating offers bargaining model of incomplete information by Rubinstein (1985). The additional feature is the option for the uninformed player to search for information in periods when he is supposed to make offers.

Two players bargain over the division of a pie of size one. The set of feasible agreements is

$$X = \{(x, 1-x) : 0 \leq x \leq 1\},$$

where x is the proportion of the pie Player 1 gets from the agreement. Let $T = \{0, 1, 2, ...\}$ be the set of times at which players move. Without information collection, Player 1 makes offers in even periods ($t = 0, 2, 4, ...$) and responds to Player 2's offers in odd periods ($t = 1, 3, ...$), and vice versa for Player 2. Let (x, t) denote the outcome of agreement x being reached in period t. The outcome of perpetual disagreement is denoted by PD. I denote the preference relations of Players 1 and 2 over $X \times T \cup \{PD\}$ by $\succcurlyeq_1$ and $\succcurlyeq_2$. Following Rubinstein (1985), I assume that the player's preferences can be represented by a utility function of the form

$$u(x)\delta^t,$$

where u is an increasing and concave function and δ can be different for the two players. Furthermore, players are expected-utility maximizers.

All aspects of the game are common knowledge, except Player 2's time preference. In particular, Player 2's preference can be either $\succcurlyeq_w$("weak") or $\succcurlyeq_s$("strong"), which is her private information. Player 1 prefers to bargain with a weak opponent. If Player 2 has preference $\succcurlyeq_w$ ($\succcurlyeq_s$), she is called Player 2_w (2_s). Player 2 is weak with probability

$$Pr(\succcurlyeq_2 = \succcurlyeq_w) = \pi, \; 0 < \pi < 1.$$

Rubinstein (1982) finds the unique subgame perfect equilibrium of the game under complete information. Let $(V_i, \hat{V}_i)$ be respectively the complete-information equilibrium divisions when Player 1 starts the bargaining and when Player 2_i starts the bargaining, where $i = w, s$. When players have fixed discounting rates, represented as $\delta_1^t x$ for Player 1 and $\delta_i^t(1-x)$ for Player 2_i, we have

$$(V_i, \hat{V}_i) = (\frac{1-\delta_i}{1-\delta\delta_i}, \delta\frac{1-\delta_i}{1-\delta\delta_i}), \qquad\qquad i = w, s.$$

The following assumptions are about the comparisons between 2_w and 2_s.

Assumption (C-1): If $x \neq 1$ and $(y, 1) \sim_w (x, 0)$, then $(y, 1) \succ_s (x, 0)$.
Assumption (C-2): $(V_s, 1) \succ_w (\hat{V}_w, 0)$.

The first assumption is that 2_s is more patient than 2_w. This assumption means that $\delta_w < \delta_s$ for the fixed-discounting-rate case. Since 2_w is more impatient than 2_s, $V_w > V_s$ and $\hat{V}_w > \hat{V}_s$. The second assumption states that Player 2_w prefers the complete-information partition between Players 1 and 2_s, even if 1 starts the bargaining and there is one period of delay. This rules out the possibility that in equilibrium 2_w sorts herself by making an offer z satisfying $(z, 0) \succcurlyeq_1 (V_w, 1)$ (and thus $(z, 0) \preccurlyeq_w (\hat{V}_w, 0)$) and $(z, 0) \succcurlyeq_w (V_s, 1)$.

Now, I introduce information collection. I assume that, in periods where Player 1 is supposed to propose, he has the option of drawing an exogenously given signal about his opponent's type.[3] Table 1 describes the signal. The numbers in the table stand for the probability of getting a certain signal given Player 2's true type. I assume $0 < \alpha < 1/2$, thus a smaller α means a more accurate signal.

Information collection is costless to Player 1 except that it delays bargaining. He gets one signal in each period if he refrains from making an offer. He can draw as many signals as he wants by not making serious offers in the subsequent periods. There is no way for Player 2 to start or restart the bargaining process unless Player 1 chooses to.[4] Furthermore, once Player 1 draws a signal, it becomes common knowledge between the two players.[5]

A *history* of the game describes not only the past proposals and responses, but also all the signals Player 1 has drawn. For simplicity, we use h^t to denote a history in period t and x^t the offer in period t, with no reference to the identity of the player to move in that history, as it should be clear from the context. I omit the formalism regarding strategies (σ_i) and beliefs (μ), except the observation that Player 1's action set now includes an element corresponding to information collection. Note that they

[3]The phrases "information collection" and "signal drawing" are interchangeable in this paper.

[4]This assumption is made to simplify discussion and in fact not needed. See Section 5.

[5]If the information collected by the uninformed player remains his private information, then it is possible for the uninformed player to use offers to signal his beliefs. This is what Feinberg and Skrzypacz (2005) find in a model with uncertainty about values, where they also show that delay can occur if the "uninformed" player has private information about his beliefs about his opponent's type. In their model, the "uninformed" player's belief is exogenously given. A thorough analysis of information collection where signals do not become public would likely make use of techniques in my paper and theirs.

Table 1: Description of the signal

Signal	2_w	2_s
W	$1-\alpha$	α
S	α	$1-\alpha$

are functions of game histories.

A sequential equilibrium of bargaining games requires that μ be Bayesian, and Players' strategies σ_1, σ_w, and σ_s be optimal given their beliefs and the other player's strategies. Furthermore, it requires that the *NDOC (Never Dissuaded Once Convinced)* property be satisfied, or, once $\mu(h) = 1$ or 0 for some history h, it remains so for all subsequent histories.[6] I adopt the refinement called *bargaining (rationalizing) sequential equilibrium* (referred to as "equilibrium" hereafter), which requires assumptions (B-1) through (B-4) be satisfied, in addition to the conditions for sequential equilibrium. They significantly reduce the multiplicity of sequential equilibrium in incomplete-information bargaining games. They are the same as those made by Rubinstein (1985) (as by Osborne and Rubinstein (1990)), with one minor exception in (B-2).

Assumption (B-1): Suppose Player 2 makes an offer in period t. If $\mu(h^{t-1}) \neq 1$, $(x^t, 1) \succ_s (x^{t-1}, 0)$, and $(x^{t-1}, 0) \succ_w (x^t, 1)$, then $\mu(h^t) = 0$.

Assumption (B-2): Suppose Player 2 makes an offer in period t. If $(x^t, 1) \succ_s (x^{t-1}, 0)$ and $(x^t, 1) \succ_w (x^{t-1}, 0)$, then $\mu(h^t) = \mu(h^{t-1})$.

Assumption (B-3): Player 1 always accepts an offer x if after rejecting he expects to reach an agreement whereby he is indifferent to x. In addition, when he is indifferent between collecting information and making offers, he chooses the latter.

Assumption (B-4): When it is Player 2's turn to make an offer, $\sigma_w(h^t) \geq \hat{V}_s$ and $\sigma_s(h^t) \geq \hat{V}_s$.

The first assumption says that if Player 2 rejects an offer by Player 1 and counteroffers in such a way that if Player 1 accepts the offer 2_w is worse off than 1's

[6]This assumption is adopted by many authors in the incomplete-information bargaining literature, Rubinstein (1985) among them. In this model, the NDOC assumption guarantees that whenever the informed player reveals her type to the uninformed player through her offers or acceptance/rejection behavior, the two players play the complete-information outcome. See Madrigal, Tan, and Werlang (1987) and Nöldeke and van Damme (1990) for a discussion of its limitations.

original offer, yet 2_s is better off, then Player 1 puts probability one on the event that Player 2 is 2_s. The second assumption says that if Player 2 rejects an offer by Player 1 and counteroffers in such a way that both 2_w and 2_s would be better off if Player 1 accepts the offer, then Player 1 cannot *change* his subjective probability that he is playing against 2_w.[7] The third assumption breaks the tie in favor of earlier agreement for Player 1 when he is indifferent. The fourth assumption limits Player 2's range of offers, that is, 2_w and 2_s cannot use offers that are rejected by Player 1 with certainty to sort themselves.

The following theorem is from Rubinstein (1985). It characterizes the equilibrium of the bargaining game without information collection. Define the pair (x^π, y^π) as

$$\begin{aligned}(x^\pi, 0) &\sim_w (y^\pi, 1),\\ (y^\pi, 0) &\sim_1 \pi(x^\pi, 1) \oplus (1-\pi)(z(x^\pi), 2),\end{aligned}$$

where $z(x)$ is defined as the z that satisfies $(x, 0) \sim_w (z, 1)$. Note that from the above, $y^\pi = z(x^\pi)$.

Theorem 0. *For a game starting with Player 1's offer: (i) If π is high enough such that $y^\pi > \hat{V}_s$, then the only bargaining sequential equilibrium outcome is $\langle (x^\pi, 0), (y^\pi, 1) \rangle$. (ii) If π is low enough such that $y^\pi < \hat{V}_s$, then the only bargaining sequential equilibrium outcome is $\langle (V_s, 0), (V_s, 0) \rangle$.*

The theorem states that for a general class of preferences, imposing certain plausible assumptions on Player 1's beliefs, the game never lasts beyond the second period in equilibrium. Rubinstein argues "it is not clear whether Player 1 would benefit from having the information about Player 2's type." In this paper, I explore what happens if Player 1 has the option to collect information about Player 2's type.

Note that (x^π, y^π) are both strictly increasing in π. The more likely Player 2 is weak, the more favorable the situation is for Player 1. As Player 1 maximizes expected value of $u(x)\delta^t$, his payoff can be represented by the following function

$$v_0(\pi) = \begin{cases} u(V_s), & \pi < \pi^*;\\ u(y^\pi)/\delta, & \pi > \pi^*.\end{cases}$$

where π^* is defined as the boundary point at which $y^\pi = \hat{V}_s$, and hence $u(V_s) = u(y^\pi)/\delta$. Thus, v_0 is increasing, and strictly increasing if $\pi > \pi^*$. Note also that

[7] This is a strengthened version of the assumption in Rubinstein's model. His assumption is that the subjective probability cannot *increase*.

$v_0(1) = u(V_w)$. Since the utility functions of the players are concave, which are necessarily continuous, v_0 is also continuous.

In the model, Player 1's beliefs will be common knowledge throughout the game. Therefore, every information set where Player 1 has the same belief about Player 2 being weak can be treated essentially the same.

3 Characterization of Equilibrium

First, I establish a lemma that characterizes Player 1's information collection behavior.

Lemma 1. *In the equilibrium, once Player 1 starts the bargaining process, he never halts it to draw additional signals. Formally, if $x_1^T \neq \phi$ for some T, then $s^t = \phi$ for all $t \geq T$. Furthermore, the equilibrium bargaining outcome must be as described in Theorem 0, with π equal to Player 1's updated belief about Player 2 being 2_w when he starts the bargaining process.*

Proof. Suppose Player 1 starts the bargaining process by offering x. There could be three types of possible responses from 2_w and 2_s: i) both reject it; ii) both accept it; iii) 2_s rejects it, and 2_w accepts it (the case in which 2_s accepts while 2_w rejects the offer is impossible, since 2_s is more patient than 2_w, and if 2_w prefers to wait, then 2_s must also prefer to wait). In case ii), the bargaining game is over. In case iii), Player 1 should be able to infer Player 2's type, then no additional signals are to be drawn. In case i), Rubinstein (1985) shows in his Proposition 4 that if 2_w and 2_s both reject the offer, then they must make the same counteroffer.[8] Player 1 should either reject or accept this counteroffer. If he accepts it, then again the bargaining game is over. If he rejects it, then the game is back to the same situation as it was when he started the bargaining process. In particular, Player 1's subjective probability that Player 2 is 2_w should not change. If Player 1 finds it profitable to draw additional signals now, he should not have started making offers two periods before. This completes the proof of the first part.

Now that Player 1 will not draw more signals once he starts the bargaining, the game is equivalent to the original bargaining game without information collection. So its rationalizing sequential equilibrium must be as specified in Theorem 0. □

[8] If the two types make different offers, then no more information collection is needed, the subgame is the same as that in which there is no information collection. So, I may apply Rubinstein's argument.

This lemma states that if Player 1 ever collects information about Player 2's type, he always does so before he starts to make offers. It lays the foundation for the rest of the paper. Player 1 obtains as many signals as is beneficial to him, and then starts to make offers. Furthermore, once he starts to make offers, the bargaining outcome is described by Theorem 0. In particular, Player 1's payoff must be consistent with what Theorem 0 requires. Thus, in characterizing the equilibrium of this model, I can focus on the decision problem of Player 1 at the beginning of the bargaining game. In Section 5, I discuss the robustness of this result.

Let $p^W(\pi,\alpha)$ and $p^S(\pi,\alpha)$ be respectively Player 1's updated belief that Player 2 is weak after receiving the signal "W" and "S," given his initial belief, π, and noise of signals, α. The functions, p^W and p^S, both mapping from $[0,1]\times(0,1/2)$ to $[0,1]$, can be written as follows.

$$\begin{aligned} p^W(\pi,\alpha) &= \pi(1-\alpha)/(\pi(1-\alpha)+(1-\pi)\alpha), \\ p^S(\pi,\alpha) &= \pi\alpha/(\pi\alpha+(1-\pi)(1-\alpha)). \end{aligned}$$

Given Lemma 1, in equilibrium, Player 1's decisions follow the Bellman's equation

$$\begin{aligned} v(\pi) = \max\{v_0(\pi),\ \delta(\pi(1-\alpha)+(1-\pi)\alpha)v(p^W(\pi))+ \\ \delta(\pi\alpha+(1-\pi)(1-\alpha))v(p^S(\pi))\}. \end{aligned} \tag{1}$$

The first argument of the maximum function is Player 1's continuation payoff from starting to offer, and the second one is that from obtaining a signal. Note that the function v completely describes the uninformed player's behavior in equilbrium. For any given value of π, if the first argument is bigger than or equal to the second one, then Player 1 starts to offer, and plays according to the equilibrium specified in Theorem 0. Otherwise, Player 1 draws a signal, based on which he updates his beliefs and makes the decision again according to the Bellman's equation, and so on.

The following lemma describes properties of the functions p^W and p^S.

Lemma 2. *The functions p^W and p^S satisfy the following properties.*

1. *$p^W(\pi,\alpha)\geq\pi$, and $p^S(\pi,\alpha)\leq\pi$, with strict inequalities when $\pi\neq 0$ or 1. Furthermore, π is a convex combination of $p^W(\pi,\alpha)$ and $p^S(\pi,\alpha)$.*[9]

$$\pi = [\pi(1-\alpha)+(1-\pi)\alpha]p^W(\pi,\alpha)+[\pi\alpha+(1-\pi)(1-\alpha)]p^S(\pi,\alpha).$$

[9] By convex combination, I mean the linear (nonnegative) coefficients on the terms sum up to 1. The whole expression is not necessarily linear. This is used in a number of occasions in the rest of the paper.

2. *Both p^W and p^S are continuous and strictly increasing in π. The function p^W is decreasing in α, and strictly decreasing in α except when $\pi = 1$ or 0; and p^S is increasing in α, and strictly increasing in α except when $\pi = 1$ or 0.*

3. *Fixing α, p^W and p^S are mutually inverse functions. That is, $p^W(p^S(\pi)) = p^S(p^W(\pi)) = \pi$.*

Proof. Most of the statements are straightforward from the definition. I only provide the proof of Part 3.

$$p^W(p^S(\pi)) = \frac{\frac{\pi\alpha}{\pi\alpha+(1-\pi)(1-\alpha)}(1-\alpha)}{\frac{\pi\alpha}{\pi\alpha+(1-\pi)(1-\alpha)}(1-\alpha) + \frac{(1-\pi)(1-\alpha)}{\pi\alpha+(1-\pi)(1-\alpha)}\alpha} = \pi.$$

Similarly for $p^S(p^W(\pi))$. □

Note that Part 3 implies Player 1's belief is determined by the "net" number of "W" and "S" signals he draws. Therefore, I can construct intervals with bounds $\{\pi^k\}_{k=0}^N$, where $\pi^k = (p^S)^k(\pi^0) = (p^W)^{N-k}(\pi^N)$.

Throughout the ensuing analysis, by virtue of von Neumann-Morganstern expected utility, I normalize Player 1's utility function such that

$$u(0) = 0, \quad u(1) = 1.$$

First, I establish a lemma that describes Player 1's signal drawing behavior in equilibrium.

Lemma 3. *In equilibrium, there exist π_L and π_H in $(0, 1)$, such that if $\pi < \pi_L$ or $\pi > \pi_H$, then Player 1 does not draw any signals.*

Proof. I prove the statement by contradiction. Suppose there is no π_L satisfying the required property. Then for any $\varepsilon > 0$, there exists $\pi < \varepsilon$ such that $v(\pi) > v_0(\pi)$.

Let $M \geq 1$ be an integer large enough such that $\delta^M < u(V_s)$. By Lemma 2, $(p^W)^M$ is continuous and strictly increasing and $(p^W)^M(0) = 0$. Thus, I can find π_0 small enough, such that $(p^W)^M(\pi_0) < \pi^*$ and $v_0(\pi_0) < v(\pi_0)$. Let $\pi_j = (p^W)^j(\pi_0)$. By Part 3 of Lemma 2, $\pi_{-j} = (p^S)^j(\pi_0)$. By Part 1 of Lemma 2, we have

$$\pi_{-M} < \pi_{-M+1} < \ldots < \pi_0 < \ldots < \pi_{M-1} < \pi_M < \pi^*.$$

Hence, $v_0(\pi_m) = u(V_s)$ for all m, such that $-M \leq m \leq M$. Thus, we have

$$u(V_s) = v_0(\pi_0) < v(\pi_0) \;=\; \delta\left[\pi_0(1-\alpha) + (1-\pi_0)\alpha\right] v(\pi_1) + \delta\left[\pi_0\alpha + (1-\pi_0)(1-\alpha)\right] v(\pi_{-1}).$$

As the expression on the right-hand side is a convex combination of $\delta v(\pi_1)$ and $\delta v(\pi_{-1})$, I conclude either $\delta v(\pi_1) > u(V_s)$ or $\delta v(\pi_{-1}) > u(V_s)$ must hold. Suppose $\delta v(\pi_1) > u(V_s)$ (the case $\delta v(\pi_{-1}) > u(V_s)$ can be similarly handled). Since $v_0(\pi_1) = u(V_s)$, we must have

$$u(V_s) < \delta v(\pi_1) = \delta^2 \left[\pi_1(1-\alpha) + (1-\pi_1)\alpha\right] v(\pi_2) + \delta^2 \left[\pi_1\alpha + (1-\pi_1)(1-\alpha)\right] v(\pi_0).$$

By the same argument as above, I have either $\delta^2 v(\pi_2) > u(V_s)$ or $\delta^2 v(\pi_0) > u(V_s)$. Proceeding by induction for M steps, I will find m, such that $|m| \leq M$ and $\delta^M v(\pi_m) > u(V_s)$, which implies $v(\pi_m) > 1$ by the definition of M, an impossible statement. So there exists π_L satisfying the desired property.

To show the existence of π_H, observe that Player 1's equilibrium payoff cannot be higher than $u(V_w)$ by Lemma 1 and Theorem 0. Since v_0 is continuous, $v_0(\pi) > \delta u(V_w)$ for π is sufficiently close to 1, making it unprofitable for Player 1 to draw signals. $\square$

The intuition for the above lemma is as follows.[10] Information is valuable to the uninformed player because it enables him to extract a higher share from the weak type. If he is quite sure that his opponent is weak, then he is already receiving very high payoffs without further information collection, hence the benefit of additional information is outweighed by the cost of delay. On the other hand, if Player 1 is quite sure that his opponent is strong, then in order to take advantage of the weak type, he must receive a large number of "W" signals, which requires a significant cost of delay, again outweighing the benefit of information. Therefore, the uninformed player does not collect information when he is quite sure about the type of his opponent. Note that, however, the lemma does not imply that Player 1's signal drawing region is an interval in the middle, i.e., Player 1 collects information if and only if he is *not* sure about Player 2's type. In the next section, I identify a special case in which this is true.

Now, I show the existence and uniqueness of the equilibrium.

Theorem 1. *There exists a unique function v solving Player 1's Bellman's equation (1). Therefore, the bargaining sequential equilibrium outcome of the bargaining game exists, and is unique.*

Proof. The proof is an application of the contraction mapping theorem.[11] The equilibrium payoff function, v, is a fixed point of the functional mapping defined by (1).

[10] I am grateful to an anonymous referee for helping me clarify this intuition.

[11] Please refer to Aubin (1993) for a statement of the contraction mapping theorem.

If I can show the mapping is a contraction, then v exists and is unique. Furthermore, Player 1's equilibrium decision rule is uniquely determined by v. Therefore, the bargaining sequential equilibrium outcome is unique.

Let V be the space of functions on $[0,1]$ bounded between 0 and 1. Define the sup norm $\| \ \|$ on the space. Denote by $T : V \to V$ the function-valued mapping defined by (1), i.e., for $w \in V$,

$$\begin{aligned} Tw(\pi) = \max\{v_0(\pi),\ & \delta(\pi(1-\alpha)+(1-\pi)\alpha)w(p^W(\pi)) + \\ & \delta(\pi\alpha+(1-\pi)(1-\alpha))w(p^S(\pi))\} \end{aligned} \tag{2}$$

For future reference, we denote the second term by the function-valued mapping $C : V \to V$. The value of $Cw(\pi)$ is the payoff Player 1 gets from collecting information at π, given w is the continuation payoff function. Therefore,

$$Tw(\pi) = \max\left\{v_0(\pi), Cw(\pi)\right\}.$$

Now, we show that T is a contraction. That is, for any two functions $w_1, w_2 \in V$, where $d = \|w_1 - w_2\|$, we show $\|Tw_1 - Tw_2\| \leq \lambda d$ for some $\lambda \in (0,1)$. For any $\pi \in [0,1]$, there are three cases: (i) $Tw_1(\pi) = Tw_2(\pi) = v_0(\pi)$; (ii) $Tw_1(\pi) \neq v_0(\pi)$, $Tw_2(\pi) \neq v_0(\pi)$; (iii) $Tw_i(\pi) = v_0(\pi), Tw_j(\pi) \neq v_0(\pi), i \neq j$.

In case (i), $|Tw_1(\pi) - Tw_2(\pi)| = 0 < \delta d$.

In case (ii), by the triangle inequality, I obtain

$$\begin{aligned} |Tw_1(\pi) - Tw_2(\pi)| &\leq \delta(\pi(1-\alpha)+(1-\pi)\alpha)\left|w_1(p^W(\pi)) - w_2(p^W(\pi))\right| \\ &\quad +\delta(\pi\alpha+(1-\pi)(1-\alpha))\left|w_1(p^S(\pi)) - w_2(p^S(\pi))\right| \\ &\leq \delta d \end{aligned}$$

The last inequality sign comes from the definition of the sup norm.

In case (iii), without loss of generality assume $Tw_1(\pi) = v_0(\pi),\ Tw_2(\pi) \neq v_0(\pi)$, then

$$\delta(\pi(1-\alpha)+(1-\pi)\alpha)w_1(p^W(\pi)) + \delta(\pi\alpha+(1-\pi)(1-\alpha))w_1(p^S(\pi)) \leq v_0(\pi)$$

and

$$\delta(\pi(1-\alpha)+(1-\pi)\alpha)w_2(p^W(\pi)) + \delta(\pi\alpha+(1-\pi)(1-\alpha))w_2(p^S(\pi)) > v_0(\pi)$$

Therefore

$$\begin{aligned} |Tw_1(\pi) - Tw_2(\pi)| &= |v_0(\pi) - Tw_2(\pi)| \\ &\leq \delta(\pi(1-\alpha)+(1-\pi)\alpha)\left|w_1(p^W(\pi)) - w_2(p^W(\pi))\right| \\ &\qquad +\delta(\pi\alpha+(1-\pi)(1-\alpha))\left|w_1(p^S(\pi)) - w_2(p^S(\pi))\right| \\ &\leq \delta d \end{aligned}$$

Thus, $\|Tw_1 - Tw_2\| \leq \delta d$. Since $0 < \delta < 1$, T is a contraction. Applying the contraction mapping theorem, I conclude that T must have a unique fixed point in V. □

Theorem 1 shows that for the general preferences discussed above, there exists a unique bargaining sequential equilibrium of pure strategies. In the equilibrium, starting from the initial prior probability π_0, Player 1 keeps drawing signals and doing Bayesian updating, until he obtains a subjective probability π, at which no signal is drawn. Then he starts the bargaining process, arriving at an equilibrium outcome according to Theorem 0. An important implication of Theorem 1 is that if the mapping T is applied to any function belonging to the space V repeatedly, then in the limit the unique solution v is reached. I can then use the limit argument to prove properties of v. This kind of reasoning will be used repeatedly in the subsequent analysis. Corollary 1.1 summarizes properties of the function v.

Corollary 1.1 The function v is continuous and increasing.

Proof. In the proof, had I defined the space V to be all *continuous* functions bounded between 0 and 1, the contraction mapping theorem would still apply. Since limits of uniformly convergent sequences of continuous functions are also continuous functions, $(V, \| \ \|)$ is a complete metric space. Hence the solution obtained is the same under these two definitions of V. So v is continuous.

I prove monotonicity from the fact that the solution is unique. For any function $w \in V$, the sequence $\{T^n w\}$ converges to the solution v. In particular, we can choose w to be increasing. The first argument of the maximum function, v_0, is increasing in π. For the second argument of the maximum function, Cw, differentiating with respect to π,[12] I get

$$\begin{aligned}(Cw)'(\pi) = {} & (1-2\alpha)[w(p^W(\pi)) - w(p^S(\pi))] \\ & + [\pi(1-\alpha) + (1-\pi)\alpha] w'(p^W(\pi))(p^W)'(\pi) \ . \\ & + [\pi\alpha + (1-\pi)(1-\alpha)] w'(p^S(\pi))(p^S)'(\pi)\end{aligned}$$

By Lemma 2, $p^W(\pi) \geq p^S(\pi)$, and both p^W and p^S are increasing in π. Since w is increasing and $0 < \alpha < 1/2$, all three terms in the expression of $(Cw)'$ are nonnegative, which shows Cw is increasing in π. So Tw is increasing. Therefore for all n, $T^n w$ is increasing in π. Since v is the limit of the sequence $\{T^n w\}$, it is also increasing. □

[12]The argument here assumes differentiability. This is to simplify the analysis. The statement would still be true without the differentiability assumption.

The function v is Player 1's expected equilibrium payoff. His payoff is continuous in his belief about his opponent's type because of continuity of preferences. It is also monotonic in his belief, as intuitively Player 1 is in a more favorable position if he faces a weak opponent with higher probability.

4 Fixed Discounting Rate

In this case, players maximize the expectation of discounted value of the share they get of the pie, as opposed to some strictly concave function of the share. Player 1's preference can be described by the function $\delta^t x$. Similarly, 2_i's ($i = w, s$) can be represented by $\delta_i^t(1-x)$. For tractability, I also assume $\delta_w = \delta$. In this case,

$$V_s = \frac{1-\delta_s}{1-\delta\delta_s}, \quad V_w = \frac{1}{1+\delta}, \quad \hat{V}_s = \delta V_s, \quad \hat{V}_w = \delta V_w;$$

$$x^\pi = \frac{1-\delta^2(1-\pi)}{1+\delta}, \quad y^\pi = \frac{\delta\pi}{1+\delta}, \quad \pi^* = \frac{(1-\delta_s)(1+\delta)}{1-\delta\delta_s}.$$

Without information collection, Player 1's payoff function takes a simple form:

$$v_0(\pi) = \begin{cases} V_s = \frac{1-\delta_s}{1-\delta\delta_s} & \pi \leq \pi^*, \\ \frac{y^\pi}{\delta} = \frac{\pi}{1+\delta} & \pi > \pi^*. \end{cases} \tag{3}$$

Note that π^* is increasing in δ. The function consists of two linear sections that intersect each other at π^*:

$$w_1(\pi) = a_1\pi + b_1, \quad w_2(\pi) = a_2\pi + b_2,$$

where

$$a_1 = 0, \; b_1 = V_s, \; a_2 = \frac{1}{1+\delta}, \; b_2 = 0.$$

Theorem 2. *For the fixed discounting rate preferences as specified above, the following are true about Player 1's signal drawing decisions:*

1. *Player 1's signal drawing region is nonempty if and only if the following condition holds:*
$$\alpha < \frac{1}{2} - \frac{(1-\delta)(b_1 a_2 - a_1 b_2)}{2\delta[(a_2+b_2)-(a_1+b_1)](b_1-b_2)}; \tag{4}$$

2. *Player 1's signal drawing region is in the form of (π_L, π_H), where $0 < \pi_L < \pi_H < 1$, if his signal drawing region is nonempty.*

Proof. First, we show Part 1 of the theorem. The proof of Part 2 will be by induction, making use of the results in Part 1.

Let $w = v_0$. By Theorem 1, the equilibrium payoff function of Player 1 is the unique fixed point of the contraction mapping T. Thus, the necessary and sufficient condition for the *absence* of information collection in equilibrium is $Tw = w$, or $Cw \leq w$ (recall that Cw denotes the second argument of the maximum function in equation (2)). Conversely, the necessary and sufficient condition for there to be a nonempty information collection region in equilibrium is that there exists a region where $Tw \neq w$, or $Cw(\pi) - w(\pi) > 0$ for π in this region.

Function Cw can be written as

$$Cw(\pi) = \begin{cases} \delta(a_1\pi + b_1), & \pi \leq p^S(\pi^*), \\ \delta\left\{(1-\alpha)a_1 + \alpha a_2 + (1-2\alpha)[(a_2+b_2)-(a_1+b_1)]\right\}\pi & \\ \qquad +\delta[(1-\alpha)b_1 + \alpha b_2], & p^S(\pi^*) \leq \pi \leq p^W(\pi^*), \\ \delta(a_2\pi + b_2), & \pi \geq p^W(\pi^*). \end{cases}$$

Since $\delta < 1$, $Cw(\pi) < w(\pi)$ for $\pi \leq p^S(\pi^*)$ and $\pi \geq p^W(\pi^*)$. See Figure 1 for a graphical illustration. From the definition of w, we have $a_1 < a_2$, $b_1 > b_2$, and $a_1 + b_1 < a_2 + b_2$. Using these facts, it is straightforward to show

$$a_1 < \delta\left\{(1-\alpha)a_1 + \alpha a_2 + (1-2\alpha)[(a_2+b_2)-(a_1+b_1)]\right\} < a_2.$$

Hence, $Cw - w$ is increasing in $[p^S(\pi^*), \pi^*]$ and decreasing in $[\pi^*, p^W(\pi^*)]$. Therefore, the necessary and sufficient condition for $Cw(\pi) - w(\pi) > 0$ for some π is that it is the case for $\pi = \pi^*$, which yields (4).

Now, we proceed to the proof of Part 2. Recall that v is Player 1's *equilibrium* payoff function. Since I assume he does not draw signals when he is indifferent between doing so and making offers, Part 2 can be equivalently stated as

$$v(\pi) > w(\pi) \text{ on } (\pi_L, \pi_H).$$

Note that T is a contraction mapping and v is its fixed point. Therefore, the sequence $\left\{T^k w\right\}$ converges to v. I prove the above statement by "induction": first, for each k, $T^k w > w$ on $(\pi_1^k, \pi_{m_k-1}^k)$ for some π_1^k and $\pi_{m_k-1}^k$; second, there exists a finite K such that $T^K w = v$.

Since Player 1 does draw signals in a nonempty region in equilibrium, then as shown in the proof of Part 1, Tw consists of three linear sections. The middle one is equal to Cw, with slope and intercept being

$$\langle a_2^1, b_2^1\rangle = \langle\delta\left\{(1-\alpha)a_1 + \alpha a_2 + (1-2\alpha)[(a_2+b_2)-(a_1+b_1)]\right\}, \delta[(1-\alpha)b_1 + \alpha b_2]\rangle.$$

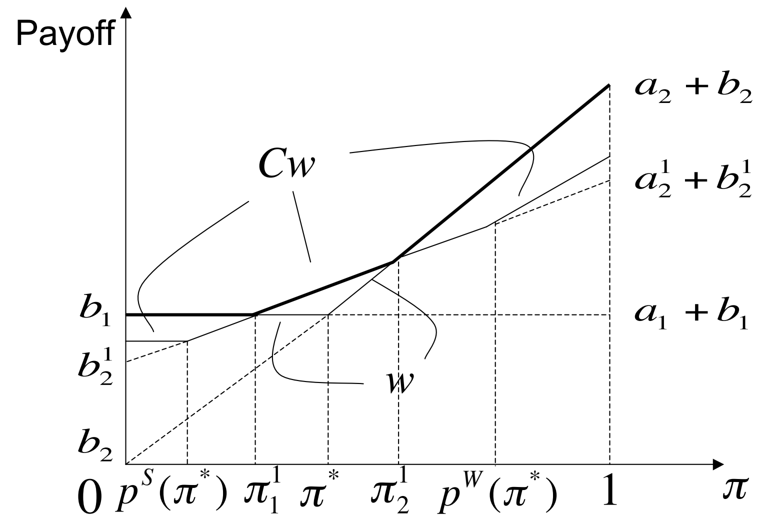

Figure 1: The mapping T in the case of fixed discounting rate. Tw consists of the three bold linear sections.

The inequality $Tw > w$ holds only on the open interval (π_1^1, π_2^1), where $p^S(\pi^*) < \pi_1^1 < \pi_2^1 < p^W(\pi^*)$. Let $a_1^1 = a_1, b_1^1 = b_1, a_3^1 = a_2$, and $b_3^1 = b_2$, then I claim that $\{a_j^1\}_{j=1}^3$ and $\{a_j^1 + b_j^1\}_{j=1}^3$ are increasing sequences, and $\{b_j^1\}_{j=1}^3$ is a decreasing sequence. This can be seen from the graph in Figure 1. Analytically, we have shown in Part 1 that the sequence $\{a_j^1\}_{j=1}^3$ is increasing. That $\{b_j^1\}_{j=1}^3$ is decreasing and $\{a_j^1 + b_j^1\}_{j=1}^3$ is increasing is because the signal-drawing region is nonempty: for $j = 1, 2$, $a_j^1\pi + b_j^1$ intersects with $a_{j+1}^1\pi + b_{j+1}^1$ on $(0, 1)$.

Now I prove by induction the following claim: for all k,

1. $T^k w$ consists of m_k linear sections intersecting with neighboring sections, which can be represented by slope and intercept pairs (a_j^k, b_j^k), $j = 1, 2, ..., m_k$, with boundary points between them being π_j^k, $j = 1, 2, ..., m_k - 1$, from left to right;

2. $a_1^k = a_1$, $b_1^k = b_1$, $a_{m_k}^k = a_2$, and $b_{m_k}^k = b_2$. Further, $\{a_j^k\}_{j=1}^{m_k}$ and $\{a_j^k + b_j^k\}_{j=1}^{m_k}$ are increasing sequences and $\{b_j^k\}_{j=1}^{m_k}$ is a decreasing sequence;

3. $T^k w > w$ holds only on the interval $(\pi_1^k, \pi_{m_k-1}^k)$, where $0 < \pi_1^k < \pi_{m_k-1}^k < 1$.

The above conditions are satisfied when $k = 1$. Assume the conditions hold for T^k, $k \geq 1$. Now, we show the conditions hold for $T^{k+1}w$. I may apply to any two linear sections of $T^k w$ the same reasoning used in the proof of Part 1. For each π, $p^S(\pi)$ and $p^W(\pi)$ lie on two linear sections of $T^k(w)$, i-th and j-th, where $i \leq j$. Thus,

$$CT^k w(\pi) = a_{ij}\pi + b_{ij},$$

where

$$a_{ij} = \delta\left\{(1-\alpha)a_i^k + \alpha a_j^k + (1-2\alpha)[(a_j^k + b_j^k) - (a_i^k + b_i^k)]\right\}, \quad b_{ij} = \delta[(1-\alpha)b_i^k + \alpha b_j^k].$$

Note that i and j weakly increases as π increases, due to induction assumptions 1 and 2. In addition, induction assumption 2 implies a_{ij} and $a_{ij} + b_{ij}$ are strictly increasing in i and j, and therefore weakly increasing in π, and b_{ij} is strictly decreasing in i and j, and therefore weakly decreasing in π. Furthermore, when $\pi \le p^S(\pi_1^k)$, $CT^k w(\pi) = \delta(a_1\pi + b_1) = \delta w(\pi) < w(\pi)$ and when $\pi \ge p^W(\pi_{m_k-1}^k)$, $CT^k w(\pi) = \delta(a_2\pi + b_2) = \delta w(\pi) < w(\pi)$. Hence

$$T^{k+1}w(\pi) = w(\pi) \text{ whenever } \pi \le p^S(\pi_1^k) \text{ or } \pi \ge p^W(\pi_{m_k-1}^k).$$

Observe that $Tw \ge w$, which by (2) implies $T^k w$ is weakly increasing in k. Since $T^k w \ge T^{k-1}w$, $CT^k w \ge CT^{k-1}w$, which implies that

$$T^{k+1}w(\pi) = CT^k w(\pi) > w(\pi) \text{ whenever } \pi \in (\pi_1^k, \pi_{m_k-1}^k).$$

Since $CT^k w(\pi)$ is continuous and increasing (as its lowest slope is $\delta a_1 = 0$) in π, there must exist a unique $\pi_1^{k+1} \in (p^S(\pi_1^k), \pi_1^k]$ and a unique $\pi_{m_{k+1}-1}^{k+1} \in [\pi_{m_k-1}^k, p^W(\pi_{m_k-1}^k))$ such that

$$CT^k w(\pi_1^{k+1}) = w(\pi_1^{k+1}) \text{ and } CT^k w(\pi_{m_{k+1}-1}^{k+1}) = w(\pi_{m_{k+1}-1}^{k+1}),$$

where m_{k+1} is the number of linear sections in $T^{k+1}w$. Observe that $a_2^{k+1} > a_1 \equiv a_1^{k+1}$ and $a_{m_{k+1}-1}^{k+1} \le \delta a_2 < a_2 \equiv a_{m_{k+1}}^{k+1}$. Thus, $T^{k+1}w$ also consists of linear sections intersecting with neighboring sections (induction assumption 1), and the sequences $\{a_j^{k+1}\}_{j=1}^{m_{k+1}}$, $\{b_j^{k+1}\}_{j=1}^{m_{k+1}}$, and $\{a_j^{k+1} + b_j^{k+1}\}_{j=1}^{m_{k+1}}$ preserve the monotonicity properties (induction assumption 2). Finally, $T^{k+1}w > w$ is true only on $(\pi_1^{k+1}, \pi_{m_{k+1}-1}^{k+1})$ (induction assumption 3). The claim is proved.

Finally, that it takes finite steps to get to v from w can be shown using the reasoning in the proof of Lemma 3. There exists M great enough, such that $\delta^M < w(\pi)$ for all π.

To conclude, $v > w$ only on an open interval (π_L, π_H). Hence, Player 1 only draws signals on (π_L, π_H). □

For the preferences considered in this section, Player 1 only draws signals between two threshold values of π. This result means that starting from any belief, Player 1, the uninformed player collects information until he is sufficiently sure about Player 2's type, so as to take advantage of the weak type as much as possible.

Condition (4) suggests that Player 1 draws signals if and only if the information source is accurate enough, other parameters being equal. It can be simplified into:

$$\alpha \quad < \quad \frac{(2\delta-1)(\delta_s-\delta)-(1-\delta_s)(1-\delta^2)}{2\delta(\delta_s-\delta)}. \tag{5}$$

But this in turn requires the following two conditions:

$$\delta_s \quad > \quad \sqrt{3}/2, \tag{6}$$

$$\delta \quad \in \quad \left(\frac{2\delta_s+1-\sqrt{4\delta_s^2-3}}{2(1+\delta_s)}, \frac{2\delta_s+1+\sqrt{4\delta_s^2-3}}{2(1+\delta_s)}\right). \tag{7}$$

This means that δ_s must be great enough and δ must be in a suitable range. That is, it can be neither too close nor too far from δ_s. If δ is too close to δ_s, then there is not too much to gain from knowing whether Player 2 is strong or weak. However, if δ is too far from δ_s, Player 1 loses too much from waiting. Conditions (5), (6), and (7) are necessary and sufficient conditions for there to be information collection in equilibrium.

Corollary 2.1. *Given the same condition as that in Theorem 2, the function v is decreasing in α at every $\pi \in [0,1]$, and the lower (upper) threshold of the signal drawing region is increasing (decreasing) in α.*

Proof. **Step 1** First I prove Tw is decreasing in α at every $\pi \in [0,1]$. Observe that the middle linear section generated by T has the following expression:

$$Cw(\pi,\alpha) = [\delta\alpha a_1 + \delta(1-\alpha)a_2 - \delta(1-2\alpha)(b_1-b_2)]\pi + \delta(1-\alpha)b_1 + \delta\alpha b_2$$

Differentiating with respect to α gives

$$\frac{\partial Cw}{\partial \alpha} = -\delta[(a_2+b_2)-(a_1+b_1)]\pi - \delta(b_1-b_2)(1-\pi) < 0,$$

because of the conditions $a_1+b_1 < a_2+b_2$ and $b_2 < b_1$. Since $Tw = \max\{v_0, Cw\}$, for every $\pi \in [0,1]$, Tw is decreasing in α no matter $v_0 \geq w$ or $v_0 < w$ at π. Note that this result does *not* depend on the fact $v_0 = w$.

Step 2 Let T_{α_1} and T_{α_2} denote respectively the mapping T when $\alpha = \alpha_1$ and $\alpha = \alpha_2$, and $\alpha_1 \leq \alpha_2$. Let v_{α_1} and v_{α_2} denote respectively the corresponding solution. By Step 1, I have $T_{\alpha_1}w \geq T_{\alpha_2}w$, hence

$$(T_{\alpha_1})^2 w \geq T_{\alpha_1}T_{\alpha_2}w.$$

Following similar argument to that in Step 1, I can prove

$$T_{\alpha_1} T_{\alpha_2} w \geq (T_{\alpha_2})^2 w.$$

Since they are generated respectively by applying T_{α_1} and T_{α_2} on $T_{\alpha_2} w$, I can use the result of Step 1. For any two linear sections of $T_{\alpha_2} w$, T_{α_1} gives a higher middle section than T_{α_2} does. It is possible that at some point π, $p^W(\pi, \alpha_1)$ is so large or $p^S(\pi, \alpha_2)$ so small (by part 2 of Lemma 2, $p^W(\pi, \alpha_1) > p^W(\pi, \alpha_2)$ and $p^S(\pi, \alpha_1) < p^S(\pi, \alpha_2)$ for all $\pi \in (0, 1)$) that $p^W(\pi, \alpha_1)$ is on a different linear section from $p^W(\pi, \alpha_2)$ or $p^S(\pi, \alpha_2)$ is on a different linear section from $p^S(\pi, \alpha_1)$. But whenever this happens, $p^W(\pi, \alpha_1)$ will be on a higher section than if the section that $p^W(\pi, \alpha_2)$ is on is extended to $p^S(\pi, \alpha_1)$. Similarly for $p^S(\pi, \alpha_1)$. For the same linear sections, T_{α_1} gives a higher middle section than T_{α_2} does. Now that $p^W(\pi, \alpha_1)$ or $p^S(\pi, \alpha_1)$ is on a higher linear section, then T_{α_1} is even higher than T_{α_2}, hence $T_{\alpha_1} T_{\alpha_2} w \geq (T_{\alpha_2})^2 w$. Now I have shown that

$$(T_{\alpha_1})^2 w \geq (T_{\alpha_2})^2 w.$$

Proceeding by induction, I can show that $(T_{\alpha_1})^k w \geq (T_{\alpha_2})^k w$ for all k if $\alpha_1 \leq \alpha_2$. Since the limits of the iteration (in fact, it is over in finite steps) are the solutions v_{α_1} and v_{α_2}, $v_{\alpha_1} \geq v_{\alpha_2}$ if $\alpha_1 \leq \alpha_2$. That is, v is decreasing in α at every $\pi \in [0, 1]$.

Recall in Part 2 of Theorem 2, the signal drawing region is an open interval in the middle. Since v is decreasing in α at every $\pi \in [0, 1]$, the lower (upper) threshold of the signal drawing region is increasing (decreasing) in α. □

The above corollary says that for the preferences considered in this section, the equilibrium has intuitive comparative statics. In general, if v_0 is continuous and piecewise linear with increasing slopes, the above theorem and corollary would apply. That v is decreasing in α at every π shows that the uninformed player is always better off having a more accurate signal about the informed player. This in turn implies that the signal drawing region is also bigger for a more accurate information source, as the uninformed player can only increase her payoff through information collection.

5 Discussion

In this paper, I consider one particular reason why agreement may be delayed in bargaining: the possibility to collect more information. I prove existence and uniqueness of equilibrium for the general case, and characterize the equilibrium in detail and perform comparative statistic analysis on the fixed-discounting-rate case.

With minor modifications, most of the results in this paper continue to hold for the fixed bargaining costs preference, i.e., that represented by the utility function $x - ct$, which is distinct from the preferences covered in this paper. The interested reader may refer to Li (2007).

Note that Lemma 1 plays a very important role in the characterization of the equilibrium. It relies on the assumption that there are only two types of the informed player. Unfortunately, it is not generally true for more than two types. Consider the following example with three types, weak, medium, and strong, in which the medium type has the same time preferences as the uninformed player. Assume the weak type has a very low discount factor and happens with probably almost equal to one. In addition, assume that if there were only the medium and strong types, the uninformed player would collect information. It is straightforward to show that the uninformed player would first use a very low offer to screen out the weak type, and collect information about the medium and strong types. The reason is that, the loss from a period of delay in reaching agreement with the weak type overwhelms the potential gain from information collection. To obtain general results about information collection behavior, it is necessary to characterize the equilibria of Rubinstein's (1985) model without information collection, which remains an open question to the best of my knowledge.

Although it is not necessary for the result that signals come in each period, they must come frequently enough in order for the result of this paper to have significance. If the information source produces signals sparsely, then it is easy to see that the uninformed party will not delay the bargaining process at all. So my result would be subject to the "Coase conjecture" critique should it be applied to the model in which the uncertainty is about valuations. It may be plausible to argue that offer-making can be done frequently, but less so to argue the same about information collection. Therefore, if the "Coase conjecture" critique is valid, the delay caused by information collection will vanish as the players are allowed to make frequent offers.

It may seem that the result depends on the uninformed player's ability to commit to not returning to the bargaining table. This is not the case. In fact, the informed player can only lure the uninformed player back to bargaining by sweetening the offer she would have otherwise made without the informed player having the information collection option. The weak type and strong type must make the same counteroffer by Rubinstein (1985). However, the strong type would not go along with this sweetened offer, since by Assumption (B-1), by separating herself from the weak type she would be able to achieve a higher payoff.

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