To Disclose or Not to Disclose: Cheap Talk with Uncertain Biases*

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Abstract. I study strategic information transmission when biases are uncertain. A perfectly informed expert advises a decision maker. The expert has biases with direction unknown to the decision maker. I show that all equilibria are of partitional form as identified by Crawford and Sobel (1982). It never benefits the decision maker or the expert to have the bias of the expert disclosed. The decision maker is better off when the bias distribution is more balanced or when the bias size is smaller.

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1 Introduction

> One can only give an unbiased opinion about things that do not interest one, which is no doubt the reason an unbiased opinion is always valueless. The man who sees both sides of a question is a man who sees absolutely nothing.
>
> Oscar Wilde (1856-1900)

People frequently have to make decisions without full information. Investors decide whether to buy or sell a stock. Legislators decide how to vote on bills. CEO's decide how to allocate budgets and personnel. In such cases, decision makers often obtain advice from experts who are better informed.

Experts rarely have the same objectives as decision makers. An investment advisor or analyst may have taken a long or short position on the stock about which she makes recommendations. A political advisor may have conservative or liberal biases, or be beholden to special interests. Therefore, experts may have incentives to distort information so as to induce decisions that are favorable to them. Decision makers are aware that experts may be biased, and so must account for this when making inferences from experts' reports.

Often, decision makers are uncertain about experts' biases. When conflicts of interest arise, does transparency about such conflicts improve communication? Should disclosure of such conflicts be mandated, or should those with possible conflicts of interest be required to recuse themselves?

News organizations adopt two kinds of rules regarding conflicts of interest. The Wall Street Journal, CBS Marketwatch, and TheStreet.com require that reporters avoid conflicts of interest altogether or recuse themselves when such conflicts arise. CNBC and Motley Fools require only that such conflicts be fully disclosed. Maria Bartiromo, the stock-market reporter and anchorwoman for CNBC, recently caused controversy when she revealed that she was holding Citigroup stocks before interviewing the company's resigning CEO, Sanford I. Weill. Amy Zelvin, a spokeswoman for CNBC, said that Ms. Bartiromo had abided by the network's policies, which require disclosure of stock ownership during on-air discussions about companies that involve more than passing mention. On the other hand, Robert M. Steele, senior faculty member and ethics group leader at the Poynter Institute, said, "Disclosure doesn't resolve a conflict of interest; all it does is reveal that a conflict exists."[1]

[1]This example is taken from McGeehan (2003).

While news organizations strive to avoid biases or to make them transparent, many lobbying organizations strive to hide their biases. They commonly use misleading names or mission statements to conceal their true interests, and to present themselves as educational, academic, and nonpartisan organizations. Water Environment Federation, "although its name evokes images of cascading mountain streams," is the sewage industry's main trade, lobby, and public relations organization.[2] American Council on Science and Health is a group partially funded by corporations like Anheuser-Busch, Giba-Geigy, Dow Chemical, etc. Its members frequently publish articles and write op-ed pieces to refute charges of cancer risks from chemicals and food additives.[3] The general public is often misinformed about the real interests of such organizations.

In this paper, I model a situation in which an expert provides advice to a decision maker. The decision maker is uncertain about the outcome of his action, which depends on some underlying state. The expert does know the state, but her preferences regarding the outcome are different from those of the decision maker. The expert's bias is her private information.[4] This model is an extension of the basic cheap talk model studied by Crawford and Sobel (1982).

I fully characterize the equilibria of the game and prove two main results. First, disclosure of biases never improves communication. In fact, neither the expert nor the decision maker prefers to have the bias of the expert disclosed. Second, the more balanced the distribution of the expert's bias, or the smaller its size, the better off is the decision maker.

Suppose that the underlying state is a number, known to the expert, that the decision maker would most like to choose a decision equal to the state, and that the expert is known to always favor a decision higher than the state by a certain amount, but not too high. Suppose further that in equilibrium, some messages from the expert induce the decision maker to choose y, and others the next larger decision $y' > y$. In order for this behavior to be optimal, the message that prompts decision y must be sent when the state is larger than y, since otherwise the decision maker would infer that the state is smaller than y and would make a smaller decision. But since

[2]The quote comes from Stauber and Rampton (1995). Related information can be found at the organization's web site: http://www.wef.org/. The web site states the organization's mission as "preserving and enhancing the global water environment."

[3]See Lutz (1996), Chapter 6, page 175. A search through Lexis-Nexis will turn up many op-ed pieces and letters to the editor written by members of the organization, with a detectable common theme.

[4]I use the word "bias" and the phrase "conflict of interest" interchangeably to describe preference misalignments between the decision maker and the expert.

the (biased) expert prefers decisions larger than the state by a certain amount, an expert who has observed a state larger than y will choose to send a message inducing decision y only if the next larger decision y' is in fact much larger than y. As a result, the decision maker can only choose relatively few actions that are spread far apart. This prevents the decision maker from choosing actions that are finely tuned to the underlying state.

Now suppose the expert could either have a positive bias, preferring decisions larger than the state, or a negative bias, preferring decisions smaller than the state. Again, a message inducing action y must sometimes be sent when the state is larger than y, but now this can be done by the negatively biased expert, who is all too happy to send a message inducing a smaller decision than the state. The positive-biased expert is then free to choose a message inducing action $y' > y$ when the state is larger than y, and as a result, the next larger action y' need not be so much larger than y. The equilibrium can then feature a larger number of potential decisions that are more closely packed, allowing the decision to better reflect the state. The decision maker can then achieve a higher expected payoff when the expert's bias is unknown. The more balanced the distribution of possible biases, the stronger is this effect.

The model presented in this paper is closely related to the model of Morgan and Stocken (2003) and somewhat related to that of Morris (2001). The former model incorporates a continuous state space, and assumes that the expert has perfect information about the state, as the model in this paper does. The latter employs a discrete state space, and assumes that the expert has imperfect information about the state. The similarity between their models and my own is that the decision maker is unsure of the expert's bias, and that the expert behaves strategically regardless of her bias. However, the bias distribution is skewed in one direction in their models – there are "good" advisors who are unbiased and "bad" advisors who have a non-zero bias in one direction. Thus, "bad" advisors always hurt "good" advisors' abilities to effectively communicate to the decision maker. However, models with a skewed bias distribution fail to capture the mitigating effect of the existence of opposite biases which is the focus of this paper.[5]

From a purely theoretical point of view, this paper extends the model of Crawford and Sobel (1982) (CS henceforth) to study cheap talk when the the expert's bias is uncertain. Unlike Morgan and Stocken (2003), I allow uncertainty about the direction

[5]As I prepared this manuscript, I also became aware of an independent paper by Dimitrakas and Sarafidis (2005). They characterize cheap talk equilibria with uncertain biases, when the bias value is allowed to be distributed on a continuum. However, they do not focus on comparisons between disclosure and nondisclosure, as I do.

of the expert's bias. I fully characterize all equilibria and perform a comparative static analysis; this has not been done for cheap talk models with uncertain biases. It is worth pointing out that this paper benefits significantly from CS and Morgan and Stocken (2003) in its methods of proof.

Bénabou and Laroque (1992) and Sobel (1985) also consider uncertainty about expert types, and focus on experts' reputation incentives. In their research, the "good" advisors are nonstrategic and always tell the truth, while the "bad" advisors are strategic and have incentives which directly conflict with those of the decision maker. Their models are thus very different from mine. There is also research that focuses on uncertainty about another dimension – competence of experts or accuracy of experts' information. Austen-Smith (1990), Ottaviani and Sørensen (2001), and Moscarini (2003) make contributions in this direction.

Farrell and Gibbons (1989) study the effects of the presence of different *audiences* on cheap talk, under a much different setup based on a discrete state space. They find the effect could be subversion, one-sided discipline, or mutual discipline.[6] In contrast, I study the effect of the existence of different types of *speakers*. The effect that one type of experts has on the other type is close in spirit to the "mutual discipline" effect in Farrell and Gibbons (1989).

The rest of the paper is organized as follows. In Section 2, I develop a simple model of cheap talk with uncertainty about biases. In Section 3, I fully characterize the equilibria of the game. In Section 4, I investigate how the decision maker's payoff depends on the distribution of biases, and determine the effect of mandatory disclosure policies. Finally, in Section 5, I interpret the results, relate them to examples and the literature, and suggest directions for further research. All proofs that are not in the main text are collected in the Appendix.

2 The Model

An expert (E or she) gives advice to a decision maker (D or he). The decision maker makes a decision that affects both his own and the expert's payoffs. Their payoffs also depend on the value of an underlying state. The state s is a random variable

[6] *Subversion* refers to cases in which the speaker is able to communicate to one audience in private, but the presence of another audience prevents him from such communication in public. *One-Sided Discipline* means that the speaker cannot communicate to one audience in private, but the presence of another audience enables him to effectively communicate with this audience. *Mutual discipline* refers to cases in which the speaker is not able to communicate to either audience in private, but is able to do so in public.

uniformly distributed on $[0,1]$. The realization of s is observable to the expert, but not to the decision maker. The expert sends a costless message m from the message set M after observing the true state. After receiving the message, the decision maker takes action $y \in \mathbf{R}$. The utility functions of the expert and the decision maker are denoted $U^E(y,s,\beta)$ and $U^D(y,s)$, where β is the expert's *bias*. They are defined by

$$\begin{aligned} U^E(y,s,\beta) &\equiv -(y-(s+\beta))^2, \\ U^D(y,s) &\equiv -(y-s)^2. \end{aligned}$$

In state s, the decision maker's most preferred action is equal to s. If the expert has bias β, her most preferred action is $s+\beta$. If $\beta > 0$, I say that the expert's bias is positive, while if $\beta < 0$, I say that the expert's bias is negative.

The expert's bias is her private information, and is drawn from the following distribution:

$$\beta = \begin{cases} b & \text{with probability } p, \\ -b & \text{with probability } 1-p. \end{cases}$$

I call the bias distribution "more balanced" when p is closer to $\frac{1}{2}$. When $p = \frac{1}{2}$, the expected bias of the expert is zero. A smaller b, of course, corresponds to a smaller bias size. I assume without loss of generality that $p \in [\frac{1}{2}, 1]$.

Let the message space M be $[0,1]$. This is not a real restriction since it is rich enough for the expert to reveal all her private information.

A strategy for an expert with bias β can then be characterized by the function $\mu_\beta : [0,1] \to [0,1]$. Let $P(s|m)$ be the belief of the decision maker about the underlying state when he receives the message m. Let $y(m)$ be the action taken by the decision maker if he receives message m. Let V^D be the expected utility of the decision maker in a strategy profile. I shall later on add arguments and/or subscripts to V^D to indicate its dependence on different parameters.

The solution concept I adopt here is *Perfect Bayesian Equilibrium.* This requires that:

E1. The decision maker's beliefs, $P(\cdot|m)$, be formed using Bayes' rule for any message m whenever possible;[7]

[7]To be precise, I need $P(\cdot|m)$ to be the regular conditional probability defined by the joint distribution of m and s. This more general definition is needed for cases when the joint distribution function of m and s is neither continuous nor discrete. See Durrett (1996) for a discussion of regular conditional probabilities.

E2. The decision maker's actions, $y(m)$, maximize his expected utility

$$\int_{[0,1]} P(s|m)U^D(y,s)\,ds$$

for all m;

E3. The expert's messages, $\mu_\beta(s)$, maximize her utility $U^E(y(m),s,\beta)$ for all s among all $m \in [0,1]$.

When describing equilibria in the rest of the paper, I omit the description of beliefs. Since the decision maker can interpret any unsent message as if it was one of the messages that *are* sent in equilibrium, beliefs about unsent messages can always be specified so that they do not disrupt E2 and E3. In addition, I assume (without loss of generality) that if messages m and m' are sent in equilibrium and induce the same action, then $m = m'$.

I consider only pure strategies for the expert.[8]

As in all cheap talk models, with or without uncertainty about biases, there is always a babbling equilibrium. In such an equilibrium, there is only one equilibrium message and action. The expert sends this message regardless of the state she observes or her own bias. The decision maker takes the action $\frac{1}{2}$ no matter what message he receives. No information is transmitted in this equilibrium.

I use the word "informative" to describe strategy profiles that give the decision maker higher payoffs than his payoff in the babbling equilibrium. A strategy profile is "more informative" than another if the former gives the decision maker higher expected utility. In cheap talk games, it is not unreasonable to expect to observe informative equilibria when they exist. In fact, there is experimental evidence (Blume, DeJong, Kim, and Sprinkle (1998)) to support this claim. A recent paper by Kartik (2003) provides a justification in the form of an equilibrium selection criterion. Consequently, when making welfare comparisons, I focus on the most informative equilibrium.

[8]In fact, this is without loss of generality, in terms of players' expected payoffs. Because each action corresponds to only one message, if an expert mixes between two reports, it must be that the two reports induce actions between which the expert is indifferent. Ties between any two actions happen only in one state for an expert of any type. It is shown below that all equilibria include only a finite number of actions, and the argument does not depend on the exclusion of mixed strategies. Therefore, the points at which ties happen have measure zero, hence do not affect expected payoffs.

3 Equilibrium

In this section, I first establish two useful facts resulting from the players' quadratic utility functions. Then, I describe the equilibrium with full disclosure. Last, I characterize the equilibria under no disclosure.

3.1 Two Useful Facts

The first fact describes how a player's ranking of two actions depends on the underlying state.

Lemma 1. *For any two actions y and y', $y < y'$, a player of bias β prefers y to y' if and only if $s \leq \frac{y+y'}{2} - \beta$.*

It is a straightforward implication of the quadratic loss preferences.[9] Therefore, the proof is omitted. Lemma 1 implies that the states in which an expert (weakly) prefers an action y to all other actions must form a closed interval when such states exist. This is because arbitrary intersections of closed intervals remain closed intervals when nonempty.

The second fact is a characterization of the decision maker's optimal action when receiving any message.

Lemma 2. *The decision maker's optimal action given any message m in equilibrium is equal to the conditional expectation $E(s|m)$.*

Proof. Any other decision rule $\tilde{y}(m)$ can be shown to add to the expected squared distance, and to reduce expected utility of the decision maker. That is,

$$\begin{aligned} E(U^D(\tilde{y}(m), s)|m) &= -E((E(s|m) - s)^2|m) - E((\tilde{y}(m) - E(s|m))^2|m) \\ &\leq E(U^D(E(s|m), s)|m). \end{aligned} \qquad \square$$

Lemma 2 says that the decision maker simply takes the action that is equal to the expected value of the underlying state. In doing this, he has taken into account possible misrepresentation by the expert due to her bias.

Note that both of these facts are true regardless of the state distribution.

[9] In general, Lemma 1 is true as long as the expert's preferences are strictly concave in y, $U^E_{12} > 0$, $U^E_{13} > 0$ and satisfy a symmetry condition: $U^D(y, s, \beta) = U^D(y', s', \beta')$ whenever $|y - (s + \beta)| = |y' - (s' + \beta')|$.

3.2 Full Disclosure of Biases.

When the decision maker learns that the expert's bias is b, the game is identical to that of CS. CS fully characterizes the equilibria when the expert's bias is common knowledge. The set of equilibria is characterized by a partition of the interval $[0,1]$ with the boundary points $\{\alpha_i\}_{i=0}^n$ satisfying

$$\begin{aligned} \alpha_0 &= 0 \\ \alpha_{i+1} - \alpha_i &= \alpha_i - \alpha_{i-1} + 4b \quad \text{for } i = 1, 2, ..., n-1 \\ \alpha_n &= 1, \end{aligned} \tag{1}$$

for each $n \in \{1, 2, ..., N(b)\}$, the number of partition elements in an equilibrium. The largest number of partition elements, $N(b)$, is the largest integer n that satisfies

$$2n(n-1)b < 1.$$

In each interval (α_{i-1}, α_i) the expert sends a message denoted by m_i. Upon receiving m_i, the decision maker takes action $y_i = \frac{\alpha_{i-1}+\alpha_i}{2}$, the conditional expectation of s. The α_i $(i = 1, ..., n-1)$ are such that the expert is indifferent between action y_i and y_{i+1} in state α_i. Lemma 1 implies $\alpha_i = \frac{y_i+y_{i+1}}{2} - b$. This is how (1) is obtained. For each n, the boundary points are uniquely determined.

It is straightforward to see that under common knowledge of bias, the set of equilibria with bias $-b$ consists exactly of equilibria that are mirror images of equilibria with bias b. Therefore, the most informative equilibrium under both cases is the same. Hence, when the bias of the expert is fully disclosed, the decision maker's highest utility is the same as his highest utility when $p = 1$.

It is worth noting that when biases are common knowledge, no information is transmitted when $b \geq \frac{1}{4}$. That is, $N(b) = 1$ for $b \geq \frac{1}{4}$. I will later compare this threshold with the threshold in the case where biases are unknown to the decision maker.

3.3 Equilibria with Uncertainty about Bias

I focus on informative equilibrium in the analysis. In order for informative equilibrium to exist, however, the expert's bias must not be too extreme.

Lemma 3. *There is no more than one action in equilibrium if $b \geq \frac{1}{2}$.*

Proof. Suppose there are at least two actions in equilibrium. Since $y(m) = E(s|m)$, I have $E(y(m)) = E(E(s|m)) = E(s) = \frac{1}{2}$. Let y, y' be the maximum and minimum of actions taken in equilibrium respectively.[10] It must be the case that $y < \frac{1}{2} < y'$. The action y' is the most preferred action by an expert of bias b if $s \geq \max\{y' - b, 0\}$. On the other hand, using Lemma 1, the message corresponding to action y' could only be sent by an expert of bias $-b$ if

$$\frac{y + y'}{2} + b \leq 1.$$

If this condition does not hold, then[11]

$$y' \leq \frac{1 + \max\{y' - b, 0\}}{2} = \begin{cases} \frac{1}{2} & \text{if } y' \leq b; \\ \frac{y'}{2} + \frac{1-b}{2} & \text{if } y' > b. \end{cases}$$

In either case of the above equation, there is a contradiction since $b \geq \frac{1}{2}$ and $y' > \frac{1}{2}$. If the condition $\frac{y+y'}{2} + b \leq 1$ does hold, then since $b \geq \frac{1}{2}$, I have

$$\frac{y + y'}{2} - b \leq 0.$$

By Lemma 1, the message corresponding to y is never sent by an expert of bias b. A contradiction similar to the one for y' above can be derived for y. So we can never have more than one action in equilibrium when $b \geq \frac{1}{2}$. □

This Lemma says when the expert's bias is large in magnitude, babbling becomes the unique equilibrium. Intuitively, since the bias is large, whenever there are two actions on opposite sides of $\frac{1}{2}$, at least one type of experts strictly prefers one action to the other in all states. Thus, one of the messages is only sent by one type. However, biases are so large that after correction, the decision maker would not want to take the corresponding action. This generates a contradiction.

This result can be contrasted with that of Morgan and Stocken (2003), where experts are either unbiased or have positive bias b. There, even when b is large, the unbiased expert can still reveal the value of the state, as long as the realized state is below a threshold (see Proposition 2, Morgan and Stocken (2003)). Furthermore, this threshold is independent of b as long as b is large enough, and converges to 1 as the probability of the expert's being biased converges to zero. The reason for the existence of such "semi-revealing equilibria" is that an expert with a large positive

[10] When the maximum and/or the minimum do not exist, I can use the supremum and/or infimum instead. Limiting arguments can be used to reach the same conclusion by the same reasoning. This applies to below proofs as well.

[11] The inequality sign appears because there may exist $s \leq y' - b$ in which an expert with bias b sends the message corresponding to y'.

bias does not want to send a message that induces a relatively low action, which enables an unbiased expert to reveal low states.

On the other hand, the threshold in Lemma 3, $\frac{1}{2}$, is larger than that in the full disclosure case, $\frac{1}{4}$. In fact, as long as $b < \frac{1}{2}$, informative equilibria do exist for all $p \in [\frac{1}{2}, 1)$. Allowing uncertainty about biases expands the set of bias sizes for which informative communication is possible.

By Lemma 3, there exist informative equilibria only if $b < \frac{1}{2}$. So I consider only $b \in (0, \frac{1}{2})$ throughout the rest of the paper.

First, I establish a result describing the expert's behavior in equilibrium. Morgan and Stocken (2003) assumed such behavior when they discuss partitional equilibria, but it is an implication of the model here.

Lemma 4. *In any informative equilibrium of the game, there is no message that is sent by only one type of expert. In other words, the decision maker can never infer with certainty the expert's bias from her reports.*

Proof. First, any message corresponding to $y \in [b, 1-b]$ cannot be sent by only one type of expert. The reason is that an expert of bias β finds the action y strictly better than all other actions at state $s = y - \beta$.

Second, there cannot be messages corresponding to actions in $[0, b) \cup (1-b, 1]$ that are sent by only one type of expert. I consider $y \in [0, b)$ only, since the case $y \in (1-b, 1]$ is similar. Let y' be the smallest such action and y'' be the largest.

I claim that $y' = y''$, which means there can be at most one action in $[0, b)$ satisfying the above requirement. If $y'' > y'$ instead, then message y' is never sent by a type b expert since y'' is strictly preferred. However a type $-b$ expert strictly prefers y' to any other action when $s \in [0, y'+b]$; thus, $y' \geq \frac{y'+b}{2}$, contradicting the assumption that $y' \in [0, b)$.

Since the equilibrium is informative, let $\hat{y}$ be the smallest action in equilibrium that is greater than y'. Now, by Lemma 1, a type $-b$ expert strictly prefers y' to all other actions when $s < \frac{y'+\hat{y}}{2} + b$. By our assumption, the message corresponding to y' is only sent by type $-b$ experts. Therefore

$$y' \geq \frac{\frac{y'+\hat{y}}{2} + b}{2},$$

which implies either

$$y' \geq \frac{y'+\hat{y}}{2},$$

or

$$y' \geq b.$$

This is impossible since it is assumed that $y' < \hat{y}$ and $y' < b$. $\square$

Now I show a result similar to Lemma 1 of CS.

Lemma 5. *If $p \in (1/2, 1]$, there is no equilibrium in which an infinite number of possible actions are taken .*

The intuition behind the above lemma is as follows. If actions are arbitrarily close to one another in equilibrium, the expert sends the message that induces action y if and only if s is arbitrarily close to $y - b$ when she has positive bias, and if and only if s is arbitrarily close to $y + b$ when she has negative bias. However, the decision maker bases his action upon the expected value of the underlying state. Since $p \neq \frac{1}{2}$, the distorting effects of the two types cannot exactly cancel each other. Thus, the expected value is different from y, which causes a contradiction. On the other hand, when $p = \frac{1}{2}$, there does exist an equilibrium in which there are infinitely many actions in equilibrium, as shown in Lemma 6.[12]

Lemma 5 establishes that there are only a finite number of actions in equilibrium if $p > \frac{1}{2}$ and $b > 0$. In particular, this means that there do not exist fully revealing equilibria or semi-revealing equilibria. There can only be partitional equilibria in this game.

I list the equilibrium actions in ascending order: $\{y_1, ..., y_n\}$, and label the messages that correspond to these actions as $\{m_1, ..., m_n\}$. Let

$$a_i = \frac{y_i + y_{i+1}}{2} \qquad i = 1, ..., n-1. \tag{2}$$

This is the average of adjacent actions. It is useful for defining the boundary points of the partition for each type. For notational convenience I also define a_i^β as follows:

$$\begin{aligned} a_0^\beta &= 0, \\ a_i^\beta &= a_i - \beta \qquad \text{for } i = 2, ..., n-1, \\ a_n^\beta &= 1. \end{aligned} \tag{3}$$

Lemma 1 implies that the message m_i is sent if $s \in [a_{i-1}^\beta, a_i^\beta]$ for bias β. For any n, each type partitions the interval $[0, 1]$ into n elements, and sends the message m_i only in the i-th element. The decision maker takes the action that is equal to the

[12] If I generalize the model to allow the positive bias and the negative bias to be of different magnitudes, then what matters in the proof is that the expected value of the bias is nonzero.

conditional expected value of the state, based on Bayesian beliefs. Formally,

$$\begin{aligned} y_1 &= \frac{a_1}{2} + \frac{b}{2} \cdot \frac{(1-2p)a_1 + b}{a_1 + (1-2p)b} = a_1 - \frac{1}{2}\delta(a_1, p, b) \\ y_i &= \frac{a_{i-1} + a_i}{2} + b(1-2p) \qquad i = 2, ..., n-1 \\ y_n &= \frac{a_{n-1} + 1}{2} + \frac{b}{2} \cdot \frac{(1-2p)(1-a_{n-1}) - b}{(1-a_{n-1}) - (1-2p)b} = a_{n-1} - \frac{1}{2}\delta(a_{n-1} - 1, p, b) \end{aligned} \tag{4}$$

In the above equation,

$$\delta(a, p, b) \equiv \frac{a^2 - b^2}{a + (1-2p)b} \equiv a - b \cdot \frac{(1-2p)a + b}{a + (1-2p)b}.$$

Such an equilibrium requires that $a_1 \geq b$ and $a_{n-1} \leq 1 - b$.[13] The following theorem fully characterizes all the equilibria of the game (except the one specified in Lemma 6, which can be obtained as the limit of partitional equilibria as n goes to infinity).[14]

Theorem 1. *Suppose $p \in [\frac{1}{2}, 1]$ and $b \in (0, \frac{1}{2})$. For $p \in (\frac{1}{2}, 1]$, there exists some positive integer $N(p, b)$ such that for each $n = 1, 2, ..., N(p, b)$ there exists a unique solution, $(\{y_i\}_{i=1}^n, \{a_i\}_{i=1}^n)$, to (2) and (4). The solution corresponds to a unique partition equilibrium with n partition elements as described in part 1 below. Furthermore, these are the only equilibria. When $p = \frac{1}{2}$, a unique solution, $(\{y_i\}_{i=1}^n, \{a_i\}_{i=1}^n)$, to (2) and (4) exists for each $n \in \mathbf{N}$. Again, the solution corresponds to a unique partition equilibrium with n partition elements as described in part 1 below.*

1. *Let $\{m_1, .., m_n\}$ be a set of distinct messages. The equilibrium with n partition elements can be characterized by:*

   a. *The expert sends message m_i if $s \in [a_{i-1}^\beta, a_i^\beta)$, where a_i^β is defined by (3).*

   b. *The decision maker takes action y_i upon receiving message m_i.*

2. *For $p \neq 1$, the number $N(p, b)$ is the largest integer n satisfying*

$$\frac{(b - 2b(n-2)^2(1-2p) - 1)^2 - b^2}{b - 2b(n-2)^2(1-2p) - 1 + b(1-2p)} - 4b(n-2)(1-2p) \leq 0, \tag{5}$$

[13] Lemma 11 in the Appendix shows that the former implies the latter when $p \geq \frac{1}{2}$.

[14] This Theorem is very similar to Proposition 3 in Morgan and Stocken (2003). However, there are two differences. First, the statement of my result does not need the qualification "suppose the decision maker cannot infer the expert's bias," which is a consequence of Lemma 4. Second, I show (in Lemma 5) that there cannot be any non-partitional equilibrium for $p \neq \frac{1}{2}$. These observations highlight the differences between models with a two-sided bias distribution and those with a one-sided distribution.

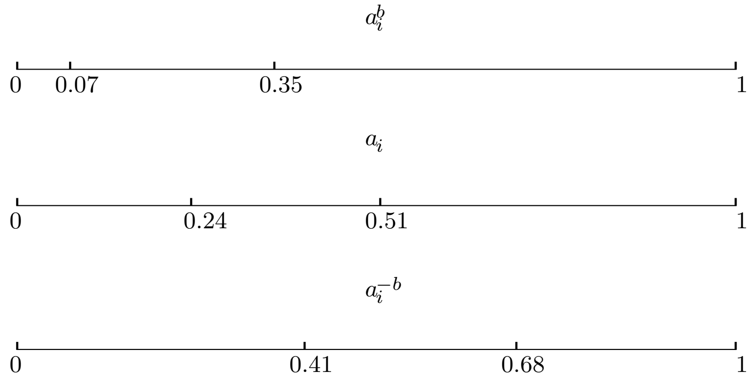

Figure 1: The 3-element equilibrium when $b = \frac{1}{6}$ and $p = \frac{2}{3}$. Each type of expert partitions the whole interval into three elements. In the i-th element, the expert sends the message that corresponds to action y_i, where $y_1 = 0.16$, $y_2 = 0.32$, and $y_3 = 0.70$.

among all n that satisfy

$$b - 2b(n-2)^2(1-2p) < 1 - b.$$

For $p = 1$, the number $N(1, b)$ is the largest integer satisfying

$$2n(n-1)b \leq 1.$$

Theorem 1 confirms that in this model, the communication equilibrium is of partitional form as identified by CS. A positive-biased expert has boundary points that are exactly $2b$ less than those of a negative-biased expert. Equation (4) reflects the fact that the action taken by the decision maker after receiving any message is equal to the conditional expectation of s. Figure 1 illustrates the equilibrium with 3 partition elements for $p = \frac{2}{3}$ and $b = \frac{1}{6}$.[15] In this equilibrium $a_1 = 0.24$ and $a_2 = 0.51$. The boundary points for an expert with bias β is a_i^β, as defined in (3).

Adding up adjacent equations in (4), I obtain

$$\begin{aligned} a_2 - a_1 &= \delta(a_1, p, b) - 2b(1-2p) \\ a_i - a_{i-1} &= a_{i-1} - a_{i-2} - 4b(1-2p) \qquad i = 3, ..., n-1, \quad (6) \\ -\delta(a_{n-1} - 1, p, b) + 2b(1-2p) &= a_{n-1} - a_{n-2} \end{aligned}$$

The condition to determine $N(p, b)$ is obtained by setting $a_1 = b$. I need to ensure that a_{n-1} does not become too large. Suppose I consider only the first $n-2$

[15]The differences between a_2 and a_1 and those between a_2^β and a_1^β should be the same. Due to rounding, the numbers in Figure 1 do not exactly satisfy this.

equations in (6). As shown in the proof of the Theorem, fixing all parameters, if the initial value a_1 increases, all boundary points will be shifted to the right, and each step $a_i - a_{i-1}$ becomes larger. However, in order for $\{a_i\}_{i=0}^n$ to be part of an equilibrium, a_{n-1} and a_{n-2} have to satisfy the last equation in (6). The left hand side is decreasing in a_{n-1} and hence decreasing in a_1, but the right hand side is increasing in a_1. Therefore, in order for there to exist an a_1 to generate boundary points that satisfy (6), the left hand side of the last equation must be greater than or equal to the right hand side when $a_1 = b$.

Corollary 1.1. *Fix the number of partition elements, $n \geq 2$, and a bias value $b \in (0, \frac{1}{2})$. In equilibrium, $\frac{\partial a_i}{\partial p} < 0$ for $i = 1, ..., n-1$ at all $p \in [\frac{1}{2}, 1]$.*

The above corollary implies that all boundary points are shifted to the right as p decreases to $\frac{1}{2}$ from 1. Recall that when p is equal to one, the partition at the left end is extremely short and the partition at the right end is extremely long. Thus, one can roughly interpret the corollary as that partition elements become more even in length on average as p approaches $\frac{1}{2}$.[16] The meaning of this statement is made precise in Lemma 7 of Section 4.

Corollary 1.2. *The maximum number of partition elements, $N(p, b)$, is nonincreasing in p and nonincreasing in b.*

This means that the closer p is to $\frac{1}{2}$, and the smaller the size of the bias, the more partitional equilibria there exist. The latter statement generalizes a comparative static result of CS to the incomplete information setting. Now I look at two special cases.

Example 1. If n is fixed, as p approaches $\frac{1}{2}$ the left hand side of (5) converges to $\frac{(b-1)^2-b^2}{b-1}$. Since $b \in (0, \frac{1}{2})$, this expression is always negative. If I fix any $n \in N$, (5) is satisfied for p close enough to $\frac{1}{2}$. Thus, $N(p, b)$ goes to infinity as p approaches $\frac{1}{2}$. Also the initial point a_1 corresponding to $N(p, b)$ partition elements converges to b as p goes to $\frac{1}{2}$, since a_1 is continuous in n and p. Also, for the equilibrium of a partition with $N(p, b)$ elements, each step $a_i - a_{i-1}$ goes to zero, and a_{n-1} converges to $1 - b$. So the equilibrium with the most partition elements converges to that specified in Lemma 6 below.

Example 2. When do informative equilibria exist? Such equilibria exist whenever (5) is satisfied when the number of messages n equals 2. The condition also

[16] Of course, boundary points continues to shift to the right as p decreases below $\frac{1}{2}$. But now the lengths of partition elements become more uneven on average as p becomes smaller.

reduces to $\frac{(b-1)^2-b^2}{b-1} \leq 0$, which is true for all $b \in (0, \frac{1}{2})$. Observe that there is a discontinuity in the existence of informative equilibria at the point $p = 1$. When $p = 1$, there are no informative equilibria for $b \geq \frac{1}{4}$. However, for all $p < 1$, there exist informative equilibria for all $b < \frac{1}{2}$.

Now I consider the special case $p = \frac{1}{2}$. In fact, there exists an informative equilibrium with an infinite number of messages.[17]

Lemma 6. *When $b < \frac{1}{2}$ and $p = \frac{1}{2}$, the following strategy profile constitutes an equilibrium:*

1. *An expert of bias β's strategy satisfies*

$$\mu_\beta(s) = \begin{cases} b & \textit{if } \beta = -b, \textit{ and } s \in [0, 2b]; \\ s + \beta & \textit{if } s \in [b - \beta, 1 - b - \beta]; \\ 1 - b & \textit{if } \beta = b, \textit{ and } s \in [1 - 2b, 1]. \end{cases}$$

2. *Upon receiving message m, the decision maker takes action $y(m) = m$ for all $m \in [b, 1 - b]$. For any other message, $y(m)$ can be any action in $[b, 1 - b]$.*

The proof of the Lemma is by directly checking the incentive conditions, and is omitted here. The equilibrium is illustrated by Figure 2, where Y_β stands for the mapping from the true state to the action by the decision maker. For any action y in the interval $(b, 1 - b)$, the positive-biased expert sends the corresponding message when $s = y - b$, and the negative-biased expert sends it when $s = y + b$. It is optimal for the decision maker to take action y when he receives this message, as the expected misstatement is zero due to the balanced distribution.

It is straightforward to show that the decision maker's expected utility in this equilibrium is

$$V^D = -(1 - 2b)b^2 - \int_0^{2b} (s - b)^2 ds = -b^2 + \frac{4}{3}b^3.$$

Later on, I compare this expression with payoffs from other equilibria and models.

4 Welfare

I now study how the expected utility of the decision maker, V^D, depends on b and p in equilibrium. In an equilibrium of a partition with n elements,

$$V_n^D(p, b) = -\sum_{i=1}^{n} p \int_{a_{i-1}^b}^{a_i^b} (s - y_i)^2 ds + (1 - p) \int_{a_{i-1}^{-b}}^{a_i^{-b}} (s - y_i)^2 ds \tag{7}$$

[17] The equilibrium construction for $p = \frac{1}{2}$ also appears in de Garidel-Thoron and Ottaviani (2000).

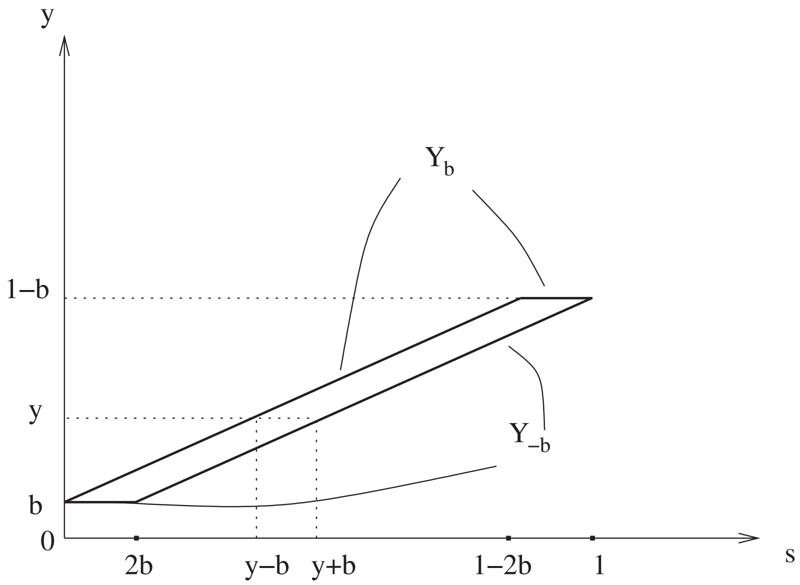

Figure 2: Equilibrium when $p = \frac{1}{2}$

Let $V_*^D(p, b)$ denote the decision maker's expected payoff in the most informative equilibrium of the game with prior $p \in [\frac{1}{2}, 1]$ and bias size $b \in (0, \frac{1}{2})$.

4.1 Comparative Statics Regarding p

The following Theorem states that the more balanced the bias distribution, the more successful the information transmission from the expert to the decision maker. It is an implication of Corollary 1.2 and Lemma 7 below.

Theorem 2. *In the cheap talk game with uncertain biases, $V_*^D(p, b)$ is strictly decreasing in p. In other words, the decision maker's equilibrium utility is increasing in the symmetry of the bias distribution.*

First, for any fixed number of partition elements, Lemma 7 implies that an equilibrium with a more balanced distribution makes the decision maker better off. Second, by Corollary 1.2, with a more balanced distribution, there exist equilibria of partitions with a larger number of elements. Thus, there are a larger set of equilibria from which to choose the most informative equilibrium. The decision maker's expected payoff in the most informative equilibrium is therefore higher. In fact, as will be shown in Lemma 8, fixing p and b, an equilibrium with a larger number of partition elements is necessarily more informative.

Lemma 7. *For any fixed $b \in (0, \frac{1}{2})$ and number of partition elements, n, $V_n^D(p, b)$ is strictly decreasing in p for all $p \in (\frac{1}{2}, 1]$.*

This Lemma says that the decision maker always strictly prefers an equilibrium in which p is smaller, given a fixed number of partition elements. A decrease in p has two effects on the expected utility of the decision maker. First, it decreases the weight placed on the expected utility from consulting the positive-biased type. The decision maker's expected utility from consulting the positive-biased type is lower than that from consulting the negative-biased type. Thus, the first effect increases the decision maker's expected utility. The second effect of a decrease in p is that all boundary points are shifted to the right. This decreases the decision maker's expected utility from consulting the negative-biased type but increases that from consulting the positive-biased type by the same amount. The latter carries more weight since $p \geq \frac{1}{2}$. Thus, the second effect also serves to increase the decision maker's overall expected utility. It should be noted that shifts in boundary points also lead to shifts in the decision maker's actions. But these actions are chosen by the decision maker to maximize his expected utility. The envelope theorem tells us that the marginal effect of such shifts on the decision maker's expected utility is zero. In conclusion, when the number of partition elements is fixed, decreasing p has the marginal effect of increasing the decision maker's expected utility.

Lemma 8. *For any $b \in (0, \frac{1}{2})$, the decision maker always prefers an equilibrium with a larger number of partition elements.*

This Lemma simply confirms the intuition that a larger number of partition elements makes communication more informative, and so benefits the decision maker. The proof of this Lemma is reminiscent of CS's proof of the analogous result under common knowledge of biases.[18]

From Lemma 8, one can conclude that the most informative equilibrium for any p and b is always the one with $N(p, b)$ partition elements. That is,

$$V_*^D(p, b) = V_{N(p,b)}^D(p, b).$$

Thus for any $b \in (0, \frac{1}{2})$, the equilibrium in Lemma 6 for the case $p = \frac{1}{2}$ gives the decision maker the highest payoff among all possible equilibria for all $p \in [0, 1]$. Furthermore, as p approaches $\frac{1}{2}$, since $N(p, b)$ goes to infinity, the most informative equilibrium approaches the equilibrium in Lemma 6.

[18]The proof is omitted here, but the interested reader can refer to the supplementary note at alcor.concordia.ca/ mingli/research/dissup.pdf.

4.2 Comparison Between Disclosure and Non-Disclosure

The following theorem is a direct implication of Theorem 2.

Theorem 3. *In the cheap talk game with uncertain biases, full disclosure of biases never benefits the decision maker.*

Proof. Note that in Section 2, I have shown that the most informative equilibrium in the case of full disclosure gives the decision maker the same payoff as that in the most informative equilibrium with $p = 1$. By Theorem 2, for any p, the decision maker's payoff in the most informative equilibrium without disclosure is always higher than the payoff under full disclosure. □

Theorem 3 is a result regarding transparency. Based on the analysis of the stylized model, I draw the rather counterintuitive conclusion that transparency about biases does not improve communication. Thus, policy makers should use caution when considering policies meant to increase transparency.

I now discuss the intuition for this result. For simplicity, I use five categories to characterize the state, the decision, and the report. They could be "extremely low," "moderately low," "average," "moderately high," or "extremely high."[19] Each happens with equal probability. Also, assume a positive-biased expert would like the action to be one notch above the true state, and vice versa for a negative-biased expert. Assume also that the expert's utility is symmetric around her most preferred action.[20]

The expert is either of positive or negative bias. When the decision maker finds out what her bias is, the factors that hinder effective communication between the expert and the decision maker exist in full force. Take, for example, a positive-biased expert. At the state "moderately low," she is indifferent between actions "extremely low" and "extremely high." She cannot credibly distinguish between any of the three highest states with the other two. All these three states are thus pooled together into one message. But given the decision maker takes an action based on the expected state, the expert would also want to pool "moderately low" with the high message. The only equilibrium then involves the expert pooling the four highest states together

[19] I also allow two actions called "excessively low" and "excessively high." The former is the most preferred action by a negative-biased expert in state "extremely low." The latter is analogous. However, these are never taken by the decision maker in equilibrium.

[20] The example I consider here has a discrete space. However, the intuition discussed here is the same as that in my model with a continuous state space.

and reporting the "extremely low" state as a separate message. Much information is lost here.

However, when there is uncertainty about the expert's bias, the decision maker can reason as follows. When the decision maker receives a moderately high message, it could be because the state is extremely high, and the expert has negative bias, or because the state is average, and the expert has positive bias. This kind of reasoning works for the middle three "moderate" messages.[21] Thus he would take the action that "matches" the meaning of the messages by taking the expected value of the state conditional on the message received. Given this reasoning by the decision maker, the expert finds it optimal to report such messages, since the action taken by the decision maker is close to her most preferred action for the bias-state combinations. This effect enables the expert and the decision maker to achieve better communication.

4.3 Comparative Statics Regarding b

The following Theorem states that a smaller bias size benefits the decision maker. It is an implication of Corollary 1.2 and Lemma 9. The proof uses an almost identical argument as that of Theorem 2 and is omitted here.

Theorem 4. *In the cheap talk game with uncertain biases, $V_*^D(p,b)$ is strictly decreasing in b.*

This Theorem confirms the analogous result of CS. In fact, the proof of Lemma 9 is an extension of the proof used by CS.

Lemma 9. *For any fixed p and number of partition elements, n, $V_n^D(p,b)$ is strictly decreasing in b.*

Lemma 9 says that the decision maker always strictly prefers an equilibrium in which b is smaller. The next result follows immediately from Corollary 1.2 and this Lemma.

It can be shown that an expert of bias β's ranking of different equilibria is exactly the same as the decision maker's regardless of her bias,[22] prior to knowing which type of expert is to be consulted. This implies that the expert prefers hiding her bias to having it disclosed. The argument is as follows. By revealed preferences, in each

[21] On the other hand, the expert still cannot separate "moderately high" and "extremely high" when she is of positive bias.

[22] The proof is a simple application of the properties of conditional and unconditional expectations, and is available from the author upon request.

equilibrium, an expert is better off when an expert of the same type is consulted than if an expert of the other type is consulted, since the set of actions to choose from is the same in the two scenarios. Observe that her expected payoff prior to learning which type is to be consulted is a convex combination of her expected payoff from herself being consulted, and that from the other type being consulted. Therefore, her expected payoff from being consulted is higher than the expected payoff evaluated prior to learning which type of expert will be consulted. By Theorem 3, the latter payoff is higher than her expected payoff when the expert's bias is disclosed. Hence, the expert prefers not to have her bias disclosed when she is the one being consulted.

5 Discussions

In this paper, I consider a simple model of cheap talk in which the direction of the expert's bias is uncertain. I find that in this scenario, it is never beneficial to the decision maker or the expert to have the bias of the expert disclosed. The decision maker also benefits when the expert's bias has a more balanced distribution or a smaller size.

One may also want to allow other distributions of bias values. For example, one may want to allow the expert to have no bias, to have a positive or negative bias of different magnitudes, to have a bias that is always positive or negative, or to have a bias drawn from a continuum of possible values. The most informative equilibrium with complete information is known. Therefore, to prove that "nondisclosure is better than disclosure" is true, it is sufficient to identify *an* equilibrium that dominates that equilibrium. However, to prove it is *not* true, it is necessary to find the most informative equilibrium under nondisclosure. The last remains an open question except for the setup in this paper. My conjecture is that a slight variation to the setup (for example, allowing the positive bias to be of slightly different magnitude from the negative bias) does not alter "the nondisclosure dominates disclosure" result, because there is a welfare gap between nondisclosure and disclosure in the current setup, as long as there is any uncertainty.[23]

[23] I have been able to construct an equilibrium similar to that in Lemma 6 when negative bias and positive bias are different in magnitude but the expected bias remains zero and the bias values satisfy certain restrictions, namely, they are not too large and not too different in size. Furthermore, the equilibrium converges to that in Lemma 6 when the positive and negative biases approach each other in absolute value. Clearly when the parameter values are close to those in Lemma 6, nondisclosure dominates disclosure.

5.1 Multiple Experts?

In this subsection, I relate the results of this paper to those of Krishna and Morgan (2001) (KM henceforth). They show that if a decision maker sequentially consults two experts of opposite biases $b_1 < 0 < b_2$, then regardless of the size of b_1, there exists a semi-revealing equilibrium. In this equilibrium, the experts reveal the value of the state when the realization of s lies in $[0, 1 - 2b_2]$, and pool the other states together. Of course, the decision maker could also switch the order of consultation, and get all information in $[-2b_1, 1]$ revealed. Thus, the decision maker can choose to ask the less biased expert last, and obtain the most informative equilibrium.

On the other hand, KM also find that sequential consultation of experts with like biases (i.e., biases in the same direction) never makes communication better than just asking the less biased expert alone. It is therefore interesting to ask the following question. Suppose that there exist both positive-biased and negative-biased experts. Two experts are randomly drawn from the population. At the beginning of the game, should the decision maker discover the biases of two experts, and then choose the best way to consult them, or should he consult them directly without discovering their biases? Here, I offer an example to show that the latter option may be preferable.

Let the distribution be $(\frac{1}{2}, \frac{1}{2})$ on $\{b, -b\} = \{\frac{1}{4}, -\frac{1}{4}\}$. This bias value is chosen so that no information is revealed when each expert is consulted alone and their biases are common knowledge. If the decision maker discovers the biases of the two experts at the beginning, then according to the results of KM, his optimal decision is as follows.

1. Consult neither of them if the biases of the two experts are the same, since no information is revealed if he consults them;

2. Consult both of them if their biases are opposite to each other, in which case information is fully revealed when $s \in [0, \frac{1}{2}]$.[24]

Each case happens with probability $\frac{1}{2}$. By simple calculation, when the decision maker chooses to discover their biases, his payoff is $-\frac{3}{64}$. If the decision maker chooses not to discover the experts' biases and asks just one of them, then according to Lemma 6, his highest payoff is

$$-(\frac{1}{4})^2 + \frac{4}{3} \times (\frac{1}{4})^3 = -\frac{1}{24},$$

[24]Without loss of generality, let the decision maker consult the negative-biased expert first.

which is strictly higher. The decision maker's highest payoff must be at least this great if he has the option of asking them both. Thus, not knowing the experts' biases benefits the decision maker. This example points to the possibility of the existence of similar results to Theorem 3 for multiple-expert scenarios.

5.2 Communication or Delegation?

The insight provided by this model is of more significant importance when communication is the only enforceable mechanism. For example, I do not consider the possibility of monetary transfers. de Garidel-Thoron and Ottaviani (2000) show that there exist full revelation equilibria when such transfers are allowed. In addition, delegation of authority is not considered here. Dessein (2002) shows that barring money transfers, when biases are common knowledge, delegation always gives the decision maker higher utility than communication, as long as communication could be informative. However, when there is uncertainty about biases, this result need not hold. In particular, full delegation gives the decision maker a payoff of $-b^2$ for sure. On the other hand, when $p = \frac{1}{2}$, the most informative communication equilibrium gives the decision maker a payoff of $-b^2 + \frac{4}{3}b^3$, which is always higher. Thus for any $b \in (0, \frac{1}{2})$, if p is close to $\frac{1}{2}$, the decision maker is better off with communication than with delegation. Therefore, although this paper is mainly concerned with scenarios in which delegation is infeasible, it also provides a reason why communication may be preferable to delegation.

In fact, a problem identified by CS is also alleviated. CS argue that if possible, under common knowledge of biases, the expert would prefer to commit to telling the truth. Without commitment, it can be shown that her expected payoff through communication with the decision maker is equal to $V^D - b^2$; with commitment to truth-telling, her expected payoff is $-b^2$. Clearly, the latter is always higher than the former since $V^D < 0$ as long as $b \neq 0$. However, when biases are uncertain, in the equilibrium of Lemma 6, the expert's payoff is

$$-\int_0^{2b} (s - (b + b))^2 \, ds + (1 - 2b) \cdot 0 = -\frac{8}{3}b^3,$$

both when her bias is b and when her bias is $-b$. Therefore, considering only $b \in (0, \frac{1}{2})$, the expert does not want to commit to truth-telling if $b \leq \frac{3}{8}$. This result is also robust to small changes in p around $\frac{1}{2}$. Hence, when the bias is small, and when the bias distribution is balanced, the expert may prefer not to commit to telling the truth. This alleviates the problem that the expert's bias seems to be self-defeating in cheap talk models.

Appendix: Proofs

Proof of Lemma 5, Page 11. Suppose to the contrary, there exist an infinite number of possible actions taken in equilibrium. Denote the set of equilibrium actions by Y^*. By the Weierstrass-Bolzano Theorem, it must have a limit point y, since it is an infinite subset of a compact set, $[0,1]$.

First, I claim $y \in [b, 1-b]$, that is, $y < b$ or $y > 1-b$ is impossible. Suppose $y < b$. Let $y' \in Y^*$ be such that $y' \neq y$ and $|y' - y| < b - y$. Such y' exists since y is a limit point of Y^*. If $y' > y$, then in any state s, an expert of bias b strictly prefers y' to each action $y'' \in Y^*$ that satisfies $y'' \neq y$ and $|y'' - y| < \frac{y'-y}{2}$. Such y'' exists again since y is a limit point of Y^*. Thus, y'' can only be sent by an expert of bias $-b$. This contradicts Lemma 4. If $y' < y$, by similar argument, y' can only be sent by an expert of bias $-b$, again contradicting Lemma 4 The argument is similar for $y > 1 - b$.

Second, I show that $y \in Y^*$. At state $s = y - b$, a positive-biased expert's most preferred action is y. Thus, inducing any action $y' \neq y$ is not optimal for her since there exists an action $y'' \in Y^*$ that is closer to y than y', due to the fact that y is a limit point of Y^*. Similarly, at state $s = y + b$, a negative-biased expert must induce action y in equilibrium. This argument in fact shows that Y^* is a closed set.

Without loss of generality, assume for any $\varepsilon > 0$, there exist an infinite number of actions in Y^*, which are *greater* than y and within ε away from y. Now, let $y < y_1 < y_2$ where $y_2 - y < \varepsilon$. By Lemma 1, an expert of bias β strictly prefers y to y_1 to y_2 if $s < \frac{y+y_1}{2} - \beta$. Similarly, he prefers y_1 to y or y_2 if $s \in (\frac{y+y_1}{2} - \beta, \frac{y+y_2}{2} - \beta)$, and y_2 to y_1 to y if $s > \frac{y+y_2}{2} - \beta$. Let us call $\frac{y+y_1}{2} - \beta$ and $\frac{y+y_2}{2} - \beta$ the cutoff points. As $\varepsilon \to 0$, $y_2 \to y$. Hence the two cutoff points both converge to $y - \beta$. Furthermore, by Lemma 1, since y_1 is neither the smallest nor the largest action in equilibrium, the states in which the positive-biased and negative-biased expert induce the action y_1 are either two equal-length intervals or two points. Therefore, y_1 converges to $p(y-b) + (1-p)(y+b) = y + (1-2p)b < y$ for $p > \frac{1}{2}$ and $b > 0$, a contradiction (since y_1 converges to y by construction). □

Proof of Theorem 1, Page 12. The argument preceding the theorem has shown that the only possible equilibria are partitional. Now I characterize the equilibrium.

By Lemma 1 the message m_i is sent if $s \in [a_{i-1}^{\beta}, a_i^{\beta}]$ for bias β. Let $\pi(m_i)$ be the decision maker's Bayesian belief about the probability that the expert has bias b, given the message m_i. Thus for $i = 1, ..., n$,

$$\pi(m_i) = \frac{p(a_i^b - a_{i-1}^b)}{p(a_i^b - a_{i-1}^b) + (1-p)(a_i^{-b} - a_{i-1}^{-b})}.$$

Substituting the definition of a_i^β into the above expression, I obtain

$$\begin{aligned}
\pi(m_1) &= \frac{p(a_1 - b)}{a_1 + (1-2p)b}, \\
\pi(m_i) &= p \qquad \text{for } i = 2, ..., n-1, \\
\pi(m_n) &= \frac{p(1 - (a_{n-1} - b))}{(1 - a_{n-1}) - (1-2p)b}.
\end{aligned}$$

Thus the partition equilibria with n elements can be described by the following difference equation:

$$y_i = \pi(m_i) \cdot \frac{a_{i-1}^b + a_i^b}{2} + (1 - \pi(m_i)) \cdot \frac{a_{i-1}^{-b} + a_i^{-b}}{2} \qquad \text{for } i = 1, ..., n.$$

Substituting the expression for $\pi(m_i)$ into the above difference equation, I obtain

$$\begin{aligned}
y_1 &= \frac{a_1}{2} + \frac{b}{2} \cdot \frac{(1-2p)a_1 + b}{a_1 + (1-2p)b} = a_1 - \frac{1}{2}\delta(a_1, p, b) \\
y_i &= \frac{a_{i-1} + a_i}{2} + b(1-2p) \qquad i = 2, ..., n-1 \\
y_n &= \frac{a_{n-1} + 1}{2} + \frac{b}{2} \cdot \frac{(1-2p)(1-a_{n-1}) - b}{(1-a_{n-1}) - (1-2p)b} = a_{n-1} - \frac{1}{2}\delta(a_{n-1} - 1, p, b)
\end{aligned}$$

which is exactly (4). In the above difference equation,

$$\delta(a, p, b) \equiv \frac{a^2 - b^2}{a + (1-2p)b} \equiv a - b \cdot \frac{(1-2p)a + b}{a + (1-2p)b}.$$

For notational convenience, I suppress the dependence of δ on p and b when there is no confusion.

The expression $\frac{1}{2}\delta(a_1, p, b)$ can be interpreted as the downward adjustment needed to calculate y_1 given a_1. This adjustment is equal to $\alpha_1/2$ for the CS case with known bias b. This simply reflects the fact that to calculate the expected value of s given that s is uniformly distributed on $[0, \alpha_1]$, one must subtract $\alpha_1/2$ from α_1, the upper bound. In this model, an additional effect exists. The expert sends message m_1 more often when her bias is $-b$ ($s \in [0, a_1 + b]$) than if her bias is b ($s \in [0, a_1 - b]$); on the other hand, the expert's bias is b with probability $p > \frac{1}{2}$. The adjustment must take the expected amount of misrepresentation into account. In the definition of δ, the two terms, a and $-b \cdot \frac{(1-2p)a+b}{a+(1-2p)b}$, account respectively for the two effects.

As a_1 goes up, the first term becomes larger. On the other hand, an increase in a_1 makes message m_1 relatively more likely to come from the expert when she is of positive bias b. Thus the downward adjustment represented by the second term also becomes larger. So δ is increasing in a_1.

As p goes up, the first term is unchanged. However, the second term becomes larger, since an increase in p means the message m_1 is more likely to be sent by an expert with positive bias b, which increases the downward adjustment needed. So δ is increasing in b.

The following fact summarizes information about the derivatives of δ with respect to a, p, and b:

Fact 1. *The derivatives of function δ satisfy:*

(i) $\delta_a(\cdot) = \frac{a^2+2(1-2p)ab+b^2}{(a+(1-2p)b)^2} = 1 + \frac{(1-(1-2p)^2)b^2}{(a+(1-2p)b)^2} \geq 1$ *for* $p \in [\frac{1}{2}, 1]$.

(ii) $\delta_p(\cdot) = \frac{2b(a^2-b^2)}{(a+(1-2p)b)^2}$. *Note* $\delta_p(\cdot) \geq 0$ *if* $|a| \geq b$ *and is strictly positive if* $|(|a) > b$.

(iii) $\delta_b(\cdot) = \frac{-2b(a+(1-2p)b)-(1-2p)(a^2-b^2)}{(a+(1-2p)b)^2}$.

The expression $-\frac{1}{2}\delta(a_{n-1}-1,p,b)$ is the upward adjustment needed to calculate y_n from a_{n-1}. Fact 1 can similarly be used to analyze properties of this expression.

Adding up adjacent equations in (4) and rearranging give

$$\begin{aligned} a_2 - a_1 &= \delta(a_1,p,b) - 2b(1-2p) \\ a_i - a_{i-1} &= a_{i-1} - a_{i-2} - 4b(1-2p) \qquad i = 3,...,n-1, \\ -\delta(a_{n-1}-1,p,b) + 2b(1-2p) &= a_{n-1} - a_{n-2} \end{aligned}$$

This is exactly Equation (6). The above difference equation can also be written in its backward form:

$$\begin{aligned} a_{n-2} - a_{n-1} &= \delta(a_{n-1}-1,p,b) - 2b(1-2p) \\ a_{n-i} - a_{n-(i-1)} &= a_{n-(i-1)} - a_{n-(i-2)} - 4b(1-2p) \qquad i = 3,...,n-1, \\ -\delta(a_1,p,b) + 2b(1-2p) &= a_1 - a_2 \end{aligned}$$

It is useful to look at partial solutions of (6); that is, boundary points that satisfy some equations in (6), but not the rest. The following Lemma considers boundary points that are defined by all equations except the last equation in (6) (and its backward version). It describes how they depend on the initial value a_1 (or for the backward version, a_{n-1}) and p.

Lemma 10. *In (6), consider all equations except the last one. Then $a_i - a_{i-1}$ is strictly increasing in $a_1 \geq b$ and p for $i = 2,...,n-1$. Hence a_i is also strictly increasing in a_1 and p. Similarly, in the backward version of (6), considering all equations but the last, $a_{n-i} - a_{n-(i-1)}$ is strictly increasing in $a_{n-1} \leq 1-b$ and p for $i = 2,...,n-1$. Hence a_{n-i} is also strictly increasing in a_{n-1} and p.*

Proof of Lemma 10. Solving (6) forward, I get

$$a_i - a_{i-1} = \delta(a_1, \cdot) - 4b(i-2)(1-2p) - 2b(1-2p),$$

for $i = 2, ..., n-1$. By Fact 1, $\delta_a > 0$, and if $a_1 \geq b$, then $\delta_p \geq 0$. It then immediately follows that $a_i - a_{i-1}$ is strictly increasing in a_1 and p for $i = 2, ..., n-1$. Since $a_i = a_1 + \sum_{j=2} (a_j - a_{j-1})$, a_i is also strictly increasing in a_1 and p for $i = 2, ..., n-1$.

The corresponding statements for the backward version can be similarly shown.

□

Equation (6) implies

$$a_i - a_1 = (i-1)\delta(a_1, \cdot) - 2b(i-1)^2(1-2p). \tag{8}$$

The backward version of (6) gives

$$a_{n-i} - a_{n-1} = (i-1)\delta(a_{n-1} - 1, \cdot) - 2b(i-1)^2(1-2p). \tag{9}$$

Adding (8) and (9) at $i = n-1$, I have

$$\delta(a_1, \cdot) + \delta(a_{n-1} - 1, \cdot) - 4b(n-2)(1-2p) = 0. \tag{10}$$

Define function λ as

$$\lambda(a, p, b, i) \equiv a + (i-1)\delta(a, p, b) - 2b(i-1)^2(1-2p).$$

For notational convenience, I suppress the dependence of λ on p and b when there is no confusion.

Given any initial value of a_1, $\lambda(a_1, i)$ gives the value of a_i according to (6) by solving it forward. On the other hand, $\lambda(a_{n-1} - 1, i)$ gives the value of a_{n-i} according to the backward version of (6), given an initial value of a_{n-1}. It is easy to show the following properties of λ:

Fact 2. *The derivatives of function λ satisfy:*

(i) $\lambda_a(\cdot) = 1 + (i-1)\delta_a(\cdot) > 0.$

(ii) $\lambda_p(\cdot) = (i-1)\delta_p(\cdot) + 4b(i-1)^2$, *which is greater than or equal to* 0 *if* $|a| \geq b$.

(iii) $\lambda_b(\cdot) = (i-1)\delta_b(\cdot) - 2(i-1)^2(1-2p).$

(iv) $\lambda_i(\cdot) = \delta(a, \cdot) - 4b(i-1)(1-2p)$, *and is nonnegative if* $\delta(a, \cdot) \geq 0$ *and* $p \in [\frac{1}{2}, 1]$.

The interpretation for the positive sign of the derivative of λ with respect to a is that all boundary points increase as the initial point increases. This is true for both forward and backward versions of (6).

Now using the definition of $\lambda(\cdot)$ in (10), I get

$$\delta(a_1, \cdot) + \delta(\lambda(a_1, n-1) - 1, \cdot) - 4b(n-2)(1-2p) = 0. \tag{11}$$

Similarly, using the definition of $\lambda(\cdot)$ in the backward version of (10) yields

$$\delta(\lambda(a_{n-1} - 1, n-1), \cdot) + \delta(a_{n-1} - 1, \cdot) - 4b(n-2)(1-2p) = 0. \tag{12}$$

The solution to Equation (11) in $[b, 1-b]$ is unique if it exists, because the left hand side is strictly increasing in a_1 by Facts 1 and 2.

Now I need to show that a unique partitional equilibrium exists for each $n \leq N(p, b)$. I first establish a useful lemma.

Lemma 11. *Fix $b > 0$, $p \in [\frac{1}{2}, 1]$, and $n \geq 2$. For any $a_1 \geq b$, if a_{n-1} satisfies (10), then $a_{n-1} \leq 1 - a_1$. Furthermore, $a_{n-1} < 1 - b$ always holds in equilibrium.*

Proof of Lemma 11. First note that the left hand side of (10) is strictly increasing in a_{n-1} since $\delta_a(\cdot) > 0$ by Fact 1. So to prove $a_{n-1} \leq 1 - a_1$, it suffices to show that if $a_{n-1} = 1 - a_1$, then the left hand side is nonnegative.

For $p \in [\frac{1}{2}, 1]$, substituting $a_{n-1} = 1 - a_1$ into (10) yields

$$\text{L.H.S. of (10)} \quad = \quad \frac{2b(1-2p)(a_1^2 - b^2)}{(1-2p)^2 b^2 - a_1^2} - 4b(n-2)(1-2p).$$

Both terms are nonnegative since $a_1 \geq b$ and $p \in [\frac{1}{2}, 1]$.[25] Thus, $a_{n-1} \leq 1 - a_1$. The first part is proved.

If $n = 2$, the above expression is equal to 0 if $a_1 = b$. This means that $a_1 = b$ and $a_{2-1} = 1 - a_1 = 1 - b$ solve (10). However, this implies $a_1 = \frac{1}{2}$, contradicting $b < \frac{1}{2}$. Thus $a_1 > b$, hence $a_{n-1} \leq 1 - a_1 < 1 - b$.

If $n \geq 3$, then the above expression is positive unless $p = \frac{1}{2}$. Thus, when $p \in (\frac{1}{2}, 1]$, $a_{n-1} < 1 - a_1 \leq 1 - b$. When $p = \frac{1}{2}$, then $a_1 > b$ since otherwise $a_i - a_{i-1} = 0$ for all i, as $\delta(a_1, p, b) = 0$ at $a_1 = b$. So $a_{n-1} \leq 1 - a_1 < 1 - b$. Thus the second part is proved. □

[25] If $p = 1$, then the first term is equal to $2b$ no matter what a_1 is. In particular, when $a_1 = b$ both the denominator and the numerator are zero, but "cancelling out" $a_1^2 - b^2$ (by taking limits) gives $2b$. On the other hand, if $p = \frac{1}{2}$, then both terms are zero.

By Lemma 11, a partitional equilibrium of size n exists if and only if there exists $a_1 \geq b$ to satisfy Equation (11). We need $a_1 \geq b$ in order for $a_1^b = a_1 - b \geq 0$.

Now I show that there exists $a_1 \geq b$ satisfying (11) if and only if the left hand side of (11) is nonpositive at $a_1 = b$. Note (11) is obtained by replacing a_{n-1} with

$$\lambda(a_1, n-1) = a_1 + (n-2)\delta(a_1) - 2b(n-2)^2(1-2p)$$

in (10). As shown in the proof of Lemma 11, the left hand side of (10) is nonnegative for $a_{n-1} \geq 1 - a_1$. When $a_1 = \frac{1}{2}$, $\lambda(a_1, n-1) \geq a_1 = 1 - a_1$. Thus the left hand side of (11) is nonnegative when $a_1 = \frac{1}{2}$. The left hand side of (11) is a continuous and strictly increasing function of a_1. Continuity and the Intermediate Value Theorem implies there exists $a_1 \in [b, \frac{1}{2}]$ solving (11) if the left hand side of (11) is nonpositive at $a_1 = b$. On the other hand, if the left hand side of (11) is positive at $a_1 = b$, then monotonicity implies that there does not exist $a_1 \in [b, \frac{1}{2}]$ solving (11).

The following facts are useful:

$$\begin{array}{lll} \delta(a,p,b) & = a+b & \text{for } p = 1; \\ \delta(a,p,b) & = 0 & \text{for } p \in [\frac{1}{2}, 1), a = b; \\ \lambda(a,p,b,i) & = b & \text{for } p = \frac{1}{2}, a = b. \end{array}$$

Before proceeding, note that the left hand side of (11) is nondecreasing in n, since the derivative is

$$\delta_a(\lambda(a_1, n-1) - 1, \cdot)\lambda_i(a_1, n-1) - 4b(1-2p) \geq 0.$$

By Facts 1 and 2 and $p \in [\frac{1}{2}, 1]$, the above expression is nonnegative, and strictly positive unless $a_1 = b$ and $p = \frac{1}{2}$. Now I substitute $a_1 = b$ into (11) and obtain for $p \in [\frac{1}{2}, 1)$,

$$\frac{(b - 2b(n-2)^2(1-2p) - 1)^2 - b^2}{b - 2b(n-2)^2(1-2p) - 1 + b(1-2p)} - 4b(n-2)(1-2p) \leq 0.$$

Additionally, I need to guarantee that

$$b - 2b(n-2)^2(1-2p) < 1 - b,$$

by the second part of Lemma 11. For $p = \frac{1}{2}$, the two equations become

$$\frac{-2b+1}{b-1} \leq 0 \Leftrightarrow b \leq \frac{1}{2}$$

and

$$b < 1 - b,$$

which are satisfied by our assumption $b \in (0, \frac{1}{2})$. These conditions do not depend on n. So there exists a partitional equilibrium for each $n \in \mathbf{N}$ if $p = \frac{1}{2}$.

For $p = 1$, the condition is $(a_1+b)+(\lambda(a_1, n-1)-1+b)-4b(n-2)(1-2)|_{a_1=b} \leq 0$, which simplifies into

$$2n(n-1)b \leq 1.$$

Since the left hand side of (11) is nondecreasing in n, when $p \in (\frac{1}{2}, 1]$, the set of numbers that satisfy (5) is the set of integers that are smaller than the largest integer that satisfies (5). Furthermore since the derivative of the left hand side of (11) with respect to a_1 is

$$\delta_a(a_1, \cdot) + \delta_a(a_{n-1} - 1, \cdot)\lambda_a(a_1, \cdot),$$

which is strictly positive for any $a_1 > b$ by Facts 1 and 2, the a_1 satisfying (11) is unique. Therefore the partitional equilibrium of size n is also unique for each $n = 1, .., N(p, b)$. □

Proof of Corollary 1.1, Page 14. First I establish that in equilibrium, a_1 is strictly decreasing in p when $p \in (\frac{1}{2}, 1)$. Applying the Implicit Function Theorem to equation (11), I obtain

$$\frac{\partial a_1}{\partial p} = -\frac{\delta_p(a_1, \cdot) + \delta_a(a_{n-1} - 1, \cdot)\lambda_p(a_1, \cdot) + \delta_p(a_{n-1} - 1, \cdot) + 8b(n-2)}{\delta_a(a_1, \cdot) + \delta_a(a_{n-1} - 1, \cdot)\lambda_a(a_1, \cdot)}.$$

By Facts 1 and 2 about the values of the derivatives of δ and λ, every term in the denominator and the numerator is nonnegative. The term

$$\delta_p(a_{n-1} - 1, \cdot)$$

is strictly positive due to $a_{n-1} < 1 - b$ by Lemma 11, and due to Part (ii) of Fact 1. Thus I conclude that $\frac{\partial a_1}{\partial p} < 0$ for all $p \in [\frac{1}{2}, 1]$.

Now I show that $\frac{\partial a_i}{\partial p} < 0$ for $i = 2, ..., n-1$ by induction. Let $1 \geq p > p' \geq \frac{1}{2}$. Let j be the smallest i such that $a_i(p) \geq a_i(p')$. Then $j \geq 2$ since it has been shown that a_1 is strictly decreasing in p. Now using (6), I have

$$\begin{aligned} a_{j+1}(p) - a_j(p) &= a_j(p) - a_{j-1}(p) - 4b(1-2p) &> a_j(p) - a_{j-1}(p) - 4b(1-2p') \\ &> a_j(p') - a_{j-1}(p') - 4b(1-2p') &= a_{j+1}(p') - a_j(p'). \end{aligned}$$

The equality signs come directly from (6). The first inequality uses the fact that $-4b(1-2p) > -4b(1-2p')$. The second inequality sign is implied by $a_j(p) \geq a_j(p')$ and $a_{j-1}(p) < a_{j-1}(p')$. With similar arguments, and by induction,

$$a_i(p) - a_{i-1}(p) > a_i(p') - a_{i-1}(p')$$

for all $i = j+1, ..., n-1$. In particular,

$$a_{n-1}(p) - a_{n-2}(p) > a_{n-1}(p') - a_{n-2}(p').$$

Summing across i, I have

$$a_{n-1}(p) - a_j(p) > a_{n-1}(p') - a_j(p').$$

This implies $a_{n-1}(p) \geq a_{n-1}(p')$ since $a_j(p) \geq a_j(p')$. Thus I have

$$-\delta(a_{n-1}(p) - 1, p, b) + 2b(1-2p) < -\delta(a_{n-1}(p') - 1, p', b) + 2b(1-2p'),$$

since $\delta_a > 0$, $\delta_p > 0$ and $2b(1-2p) < 2b(1-2p')$. By (6),

$$-\delta(a_{n-1} - 1, p, b) + 2b(1-2p) = a_{n-1} - a_{n-2}$$

must hold in equilibrium.

The last three equations cause a contradiction.[26] Hence a_i is strictly decreasing in p in equilibrium. □

Proof of Corollary 1.2, Page 14. Let $\tilde{N}(p, b)$ be the number that satisfies

$$b - 2b(n-2)^2(1-2p) < 1 - b$$

and satisfies (5) with equality. I know it is unique from the proof of Theorem 1. It suffices to show that $\tilde{N}$ is nonincreasing in p and b, since $N(p, b)$ is the largest integer smaller than or equal to $\tilde{N}(p, b)$.

The left hand side of (11) is nondecreasing in n. The derivative is

$$\delta_a(\lambda(a_1) - 1, \cdot)\lambda_n(\cdot) - 4b(1-2p) \geq 0,$$

by Fact 1. Also note that (5) is obtained by setting $a_1 = b$ in (11). Thus

$$\frac{\partial(L.H.S. \text{ of } (5))}{\partial n} = \frac{\partial(L.H.S. \text{ of } (11))}{\partial n}|_{a_1=b} \geq 0.$$

Part 1. $\tilde{N}$ is nonincreasing in p.

Note the derivative of the left hand side of (11) with respect to p is

$$\delta_p(a_1, \cdot) + \delta_a(a_{n-1} - 1, \cdot)\lambda_a(a_1, \cdot) + \delta_p(a_{n-1} - 1, \cdot) + 8b(n-2),$$

[26]Note that the same proof tactic is used in the proof of Lemma 12 on Page 33.

which by Facts 1 and 2 is strictly positive for any $a_1 \geq b$ and $a_{n-1} = \lambda(a_1, \cdot) \leq 1 - b$. I may use the implicit function theorem to obtain for $p \in [\frac{1}{2}, 1)$

$$\frac{\partial \tilde{N}}{\partial p} = -\frac{\partial(L.H.S. \text{ of } (11))/\partial p|_{a_1=b}}{\partial(L.H.S. \text{ of } (11))/\partial n|_{a_1=b}}.$$

The expression is negative since the denominator is nonnegative and the numerator is positive. Now I look at $p = 1$. Note in (5) as $p \to 1$, the inequality approaches

$$2(n-1)^2 b \leq 1.$$

In contrast, $N(1, b)$ is the largest integer that satisfies

$$2n(n-1)b \leq 1.$$

Therefore, $\lim_{p \to 1} N(p, b) \geq N(1, b)$. In fact, the largest difference between the two values is one, since $\lim_{p\to 1} \tilde{N}(p, b) - \tilde{N}(1, b) = \sqrt{\frac{1}{2b}} + 1 - (\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2}{b}}) < \frac{1}{2}$.
To summarize, $N(p, b)$ is nonincreasing in p for all $p \in [\frac{1}{2}, 1]$ and $b \in (0, \frac{1}{2})$.
Part 2. $\tilde{N}$ is nonincreasing in b.

That $\tilde{N}(1, b)$ is nonincreasing in b can be easily observed from the condition

$$2n(n-1)b \leq 1.$$

Now consider the case $p \neq 1$. Note that (5) is obtained by substituting $a_1 = b$ into (11). Also observe the facts $\delta(a_1, p, b) = 0$ and $\lambda(a_1, p, b, n-1) = 0 + b - 2b(n-2)^2(1-2p)$ at $a_1 = b$. Define

$$l(b) = b - 2b(n-2)^2(1-2p).$$

Then

$$l'(b) = 1 - 2b(n-2)^2(1-2p),$$

which is nonnegative for all $n \geq 2$. Also, $l(b) = l'(b)b$ since $l(b)$ is linear in b.

First, it is clear that $l(b) \leq 1 - b$ is required since $l(b) = \lambda(b, p, b, n-1)$ is the value of a_{n-1} when $a_1 = b$. This requires that

$$2b - 2b(n-2)^2(1-2p) \leq 1,$$

which is more likely to be satisfied by smaller b for any $n \geq 2$.

Second, (5) can be written as

$$\frac{(l(b)-1)^2 - b^2}{l(b) - 1 + b(1-2p)} - 4b(n-2)(1-2p) \leq 0.$$

Thus

$$
\begin{aligned}
& \partial(L.H.S. \text{ of } (5))/\partial b \\
= & \frac{2[(l(b)-1)l'(b)-b][l(b)-1+b(1-2p)]-[(l(b)-1)^2-b^2][l'(b)+(1-2p)]}{[l(b)-1+b(1-2p)]^2} \\
= & \frac{(l(b)-1)^2 l'(b)+[2l(b)-(l(b)-1)][l(b)-1](1-2p)-b(l(b)-1)+b-b^2(1-2p)}{[l(b)-1+b(1-2p)]^2}.
\end{aligned}
$$

In the derivation, I have used $l(b) = l'(b)b$. Each term in the numerator of the last expression is nonnegative, and some terms are strictly positive (for example, b). Thus $\partial(L.H.S. \text{ of } (5))/\partial b > 0$.

Now I use the Implicit Function Theorem to conclude

$$\frac{\partial \tilde{N}}{\partial b} = -\frac{\partial(L.H.S. \text{ of } (5))/\partial b}{\partial(L.H.S. \text{ of } (5))/\partial n} < 0. \qquad \square$$

Proof of Theorem 2, Page 16. The argument is simple. Lemma 7 states that for the same number of partition elements, the decision maker always strictly prefer the equilibrium with smaller p. On the other hand, Corollary 1.2 implies that when p is smaller, there is also a (weakly) larger set of numbers of partition elements to choose from since $N(p, b)$ is nonincreasing in p. Thus the most informative equilibrium with smaller $p \in [\frac{1}{2}, 1]$ always gives the decision maker strictly higher payoffs. $\square$

Proof of Lemma 7, Page 16. First note that y_i is the action that maximizes the decision maker's expected utility given that m_i is received, and hence must satisfy the relevant first order conditions. Thus although y_i depends on p, I may use the Envelope Theorem and ignore such dependence when taking derivatives of V^D with respect to p. Therefore

$$
\begin{aligned}
-\frac{\partial V_N^D(P,B)}{\partial p} = & \textstyle\sum_{i=1}^{n-1} p[(a_i^b - y_i)^2 - (a_i^b - y_{i+1})^2]\frac{\partial a_i^b}{\partial p} + (1-p)[(a_i^{-b} - y_i)^2 - (a_i^{-b} - y_{i+1})^2]\frac{\partial a_i^{-b}}{\partial p} \\
& + \textstyle\sum_{i=1}^{n} \int_{a_{i-1}^b}^{a_i^b} (s-y_i)^2 ds - \int_{a_{i-1}^{-b}}^{a_i^{-b}} (s-y_i)^2 ds. \qquad (13)
\end{aligned}
$$

I evaluate the two terms on the right hand side separately. Denote the first term by A_1 and the second term A_2.

First I calculate A_1. By the definitions of a_i^β and a_i, I have for all $i = 1, ..., n-1$,

$$
\begin{aligned}
a_i^b + a_i^{-b} - y_i - y_{i+1} = & \; 2a_i - (y_i + y_{i+1}) & = 0, \\
\partial a_i^b/\partial p = & \; \partial a_i^{-b}/\partial p & = \partial a_i/\partial p.
\end{aligned}
$$

Therefore

$$A_1 = \sum_{i=1}^{n-1}(2p-1)[(a_i - b - y_i)^2 - (a_i - b - y_{i+1})^2]\frac{\partial a_i}{\partial p}.$$

Note $(a_i - b - y_i) + (a_i - b - y_{i+1}) = -2b < 0$ and $y_i < a_i < y_{i+1}$. Therefore $|a_i - b - y_i| < |a_i - b - y_{i+1}|$. Since $p \geq \frac{1}{2}$ and $\frac{\partial a_i}{\partial p} < 0$ by Corollary 1.1, $A_1 \geq 0$ with equality sign only at $p = \frac{1}{2}$.

Now I consider A_2. It is useful to consider the first and the last terms of the summation ($i = 1$ and $i = n$) separately from the others. I have

$$\begin{aligned}
& \int_0^{a_1-b}(s-y_1)^2 ds - \int_0^{a_1+b}(s-y_1)^2 ds + \int_{a_{n-1}-b}^1 (s-y_n)^2 ds - \int_{a_{n-1}+b}^1 (s-y_n)^2 ds \\
= \quad & \tfrac{1}{3}[(a_1-b-y_1)^3 - (a_1+b-y_1)^3 - (a_{n-1}-b-y_n)^3 + (a_{n-1}+b-y_n)^3] \\
= \quad & \tfrac{1}{3}[-2\cdot 3(a_1-y_1)^2 b + 2\cdot 3(a_{n-1}-y_n)^2 b] \\
= \quad & \tfrac{b}{2}(-\delta(a_1)^2 + \delta(a_{n-1}-1)^2).
\end{aligned}$$

The last step uses equation (4). Now using equations (8) and (9), I conclude

$$\frac{b}{2}(-\delta(a_1)^2 + \delta(a_{n-1}-1)^2) \quad = \tfrac{b}{2}\cdot 2(a_1 - a_{n-1})\cdot 4b(1-2p) \geq 0.$$

The inequality sign is because $a_1 \leq a_{n-1}$ and $1 - 2p \leq 0$. Thus the sum of the first and last summation terms in $A2$ is nonnegative. Note that it holds as equality only if $p = \frac{1}{2}$ or $n = 2$.
The other terms ($i = 2, ..., n-1$) in A_2 can be calculated using (6):

$$\begin{aligned}
& \int_{a_{i-1}-b}^{a_i-b}(s-y_i)^2 ds - \int_{a_{i-1}+b}^{a_i+b}(s-y_i)^2 ds \\
= \quad & \tfrac{1}{3}[(a_i-b-y_i)^3 - (a_{i-1}-b-y_i)^3 + (a_i+b-y_i)^3 - (a_{i-1}+b-y_i)^3] \\
= \quad & \tfrac{1}{3}[2(a_i-y_i)^3 + 2\cdot 3(a_i-y_i)b^2 - 2(a_{i-1}-y_i)^3 - 2\cdot 3(a_{i-1}-y_i)b^2] \\
\geq \quad & 0.
\end{aligned}$$

The last inequality simply uses the fact $a_i \geq a_{i-1}$. Thus A_2 is nonnegative, and equals zero only if $p = \frac{1}{2}$ or $n = 2$.
To conclude, $\frac{\partial V_N^D(P,B)}{\partial p} \leq 0$ and is equal to zero only if $p = \frac{1}{2}$. □

The following lemma compares boundary points of n-element and $n+1$-element equilibria. It is useful in the proof of Lemma 8.

Lemma 12. *If both n-element and n+1-element equilibria exist, then for* $i = 1, ..., n-1$,

a. $a_i(n+1) < a_i(n) < a_{i+1}(n+1)$.

b. $a_{i+1}(n+1) - a_i(n+1) < a_i(n) - a_{i-1}(n)$.

Proof. **a.** I show $a_i(n+1) < a_i(n)$ by using the forward version of (6). The proof is by induction.

First I show $a_1(n+1) < a_1(n)$. Suppose to the contrary, $a_1(n+1) \geq a_1(n)$. From (6), I get

$$a_i - a_{i-1} = \delta(a_1, p, b) - 2b(1-2p) - 4b(i-2)(1-2p).$$

By Fact 1, $\delta_a > 0$. Thus if $a_1(n+1) \geq a_1(n)$, I have

$$a_i(n+1) - a_{i-1}(n+1) \geq a_i(n) - a_{i-1}(n),$$

which implies that

$$a_i(n+1) \geq a_i(n)$$

for $i = 1, ..., n-1$. Since $a_n(n+1) - a_{n-1}(n+1) = a_{n-1}(n+1) - a_{n-2}(n+1) - 4b(1-2p) \geq a_{n-1}(n+1) - a_{n-2}(n+1)$, I have

$$a_n(n+1) - a_{n-1}(n+1) \geq a_{n-1}(n) - a_{n-2}(n). \tag{14}$$

Observe that whenever $a_1 > b$ or $p \neq \frac{1}{2}$, $\delta(a_1) - 2b(1-2p)$ must be strictly positive. But for finite n, $a_1 = b$ and $p = \frac{1}{2}$ cannot both be true. So

$$a_n(n+1) - a_{n-1}(n+1) = \delta(a_1(n+1)) - 2b(1-2p) - 4b(n+1-2)(1-2p) > 0.$$

Thus $a_n(n+1) > a_{n-1}(n+1) \geq a_{n-1}(n)$. Fact 1 says $\delta_a > 0$. Therefore

$$-\delta(a_n(n+1) - 1, \cdot) + 2b(1-2p) < -\delta(a_{n-1}(n) - 1, \cdot) + 2b(1-2p). \tag{15}$$

Now since $\{a_i(n)\}$ and $\{a_i(n+1)\}$ are equilibrium boundary points, they must satisfy the following conditions:

$$-\delta(a_{n-1}(n) - 1, \cdot) + 2b(1-2p) = a_{n-1}(n) - a_{n-2}(n), \tag{16}$$

$$-\delta(a_n(n+1) - 1, \cdot) + 2b(1-2p) = a_n(n+1) - a_{n-1}(n+1). \tag{17}$$

These contradict (14) and (15).

Now I perform the second step of the induction. Assume $a_j(n+1) < a_j(n)$ for $j = 1, ..., i-1$. I want to show that $a_i(n+1) < a_i(n)$.

Suppose to the contrary, $a_i(n+1) \geq a_i(n)$. This implies that

$$a_i(n+1) - a_{i-1}(n+1) > a_i(n) - a_{i-1}(n)$$

by the induction hypothesis. By (6),

$$a_j(n+1) - a_{j-1}(n+1) > a_j(n) - a_{j-1}(n)$$

for $j = i, ..., n-1$. As in the first step, I can derive (14) and (15), which again contradict (16) and (17).

That $a_i(n) < a_{i+1}(n+1)$ can be similarly proved by using the backward version of (6).

b. That $a_{i+1}(n+1) - a_i(n+1) < a_i(n) - a_{i-1}(n)$ is an immediate consequence of Part a. To see this, suppose $a_{i+1}(n+1) - a_i(n+1) \geq a_i(n) - a_{i-1}(n)$ instead. This would imply $a_n(n+1) - a_{n-1}(n+1) \geq a_{n-1}(n) - a_{n-2}(n)$. But $a_n(n+1) > a_{n-1}(n)$. This leads to a contradiction similar to that in the proof of Part a.

Lemma 12 is thus proved. □

Proof of Lemma 9, Page 19. As in the proof of Lemma 7, I may use the Envelope Theorem and ignore the indirect dependence of V^D on b through y_i. Therefore

$$-\frac{\partial V_n^D(p,b)}{\partial b} = \sum_{i=1}^{n-1} p[(a_i^b - y_i)^2 - (a_i^b - y_{i+1})^2]\frac{\partial a_i^b}{\partial b} + (1-p)[(a_i^{-b} - y_i)^2 - (a_i^{-b} - y_{i+1})^2]\frac{\partial a_i^{-b}}{\partial b}.$$

Using the definition $a_i^\beta = a_i - \beta$ for $i = 1, ..., n$, and the fact that $a_i = \frac{y_i + y_{i+1}}{2}$, I have

$$-\frac{\partial V^D}{\partial b} = \sum_{i=1}^{n-1} [(a_i^b - y_i)^2 - (a_i^b - y_{i+1})^2][(2p-1)\frac{\partial a_i}{\partial b} - 1]. \tag{18}$$

I know from the proof of Lemma 7 that $[(a_i^b - y_i)^2 - (a_i^b - y_{i+1})^2] < 0$ for $i = 1, ..., n$. Now I show that $(2p-1)\frac{\partial a_i}{\partial b} - 1 < 0$. This implies that $\frac{\partial V^D}{\partial b} < 0$.

Let $\alpha_i \equiv p a_i^b - (1-p) a_i^{-b}$.[27] Note that $\alpha_i = (2p-1)a_i - b$ for $i = 1, ..., n-1$. The task is thus to show $\frac{\partial \alpha_i}{\partial b} < 0$ for $i = 1, ..., n-1$. I may rewrite (6) as

$$\begin{aligned}
\alpha_2 - \alpha_1 &= \alpha_1 + b + b \cdot \frac{\alpha_1}{\alpha_1 + 4p(1-p)b} + 2b(1-2p)^2 \\
\alpha_i - \alpha_{i-1} &= \alpha_{i-1} - \alpha_{i-2} + 4b(1-2p)^2 \qquad i = 3, ..., n-1, \\
\alpha_{n-1} - \alpha_{n-2} &= -[\alpha_{n-1} - (2p-1) + b + b \cdot \frac{\alpha_{n-1} - (2p-1)}{\alpha_{n-1} + 4p(1-p)b - (2p-1)} + 2b(1-2p)^2].
\end{aligned} \tag{19}$$

Considering all equations but the last, I may establish a result similar to Lemma 10. That is

1. $\alpha_i - \alpha_{i-1}$ is strictly increasing in α_1 and b for all $\alpha_1 \geq (2p-1)b - b$ and $i = 2, ..., n-1$.

2. α_i is also strictly increasing in α_1 and b for $i = 2, ..., n-1$.

[27] The symbol α_i since it has the CS model as the special case when $p = 1$, as in Section 2.

The proof is similar to that of Lemma 10 and is omitted here.

Now consider the last equation of (19). The L.H.S. is increasing in a_1 and b. The R.H.S. is strictly decreasing in a_{n-1}, since

$$\frac{\partial \frac{\alpha_{n-1}-(2p-1)}{\alpha_{n-1}+4p(1-p)b-(2p-1)}}{\partial a_{n-1}} = \frac{4p(1-p)b}{[a_{n-1}+4p(1-p)b-(2p-1)]^2} \geq 0.$$

Hence the R.H.S. is strictly decreasing in a_1 since a_{n-1} is strictly increasing in a_1 and a_{n-1} is the only channel through which the R.H.S. depends on a_1. The R.H.S. is strictly decreasing in b, since

$$\frac{\partial \frac{b[\alpha_{n-1}-(2p-1)]}{\alpha_{n-1}+4p(1-p)b-(2p-1)}}{\partial a_{n-1}} = \frac{[a_{n-1}-(2p-1)]^2+4p(1-p)b^2\frac{\partial a_{n-1}}{\partial b}}{[a_{n-1}+4p(1-p)b-(2p-1)]^2}$$

and $\frac{\partial a_{n-1}}{\partial b} > 0$. The Implicit Function Theorem thus implies that in equilibrium, a_1 decreases as b increases. That a_i $(i = 2, ..., n-1)$ is strictly decreasing in b can be shown by induction, as in the proof of Corollary 1.1.

Therefore, fixing number of elements, n, and prior, p,

$$\frac{\partial V_n^D(p,b)}{\partial b} < 0. \qquad \square$$

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