Supplementary Material to
"When Mandatory Disclosure Hurts:
Expert Advice and Conflicting Interests"[*]

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The following discussion (preceding the proof of Lemma 15) gives details of equilibrium characterization.

Let $y(a^h, \tilde{a}^h, a^l, \tilde{a}^l)$ be the action that maximizes the expected utility of the decision maker given that the high type sends the corresponding message in $[a^h, \tilde{a}^h]$ and the low type in $[a^l, \tilde{a}^l]$. Unless $a^h \geq \tilde{a}^h$ and $a^l \geq \tilde{a}^l$ both hold,[1] it is the action that solves

$$\max_y V(y, a^h, \tilde{a}^h, a^l, \tilde{a}^l) \equiv p \int_{a^h}^{\tilde{a}^h} U(y-s)\, ds + (1-p) \int_{a^l}^{\tilde{a}^l} U(y-s)\, ds.$$

Since U depends only on the difference between y and s, it is straightforward to verify that

$$y(a^h + c, \tilde{a}^h + c, a^l + c, \tilde{a}^l + c) = y(a^h, \tilde{a}^h, a^l, \tilde{a}^l) + c. \tag{S-1}$$

In other words, if all the boundary points of the intervals are translated by c, the optimal action is also translated by c.

By Lemma 1, if an action is neither the highest nor the lowest action in the set of equilibrium actions and is induced by both types of experts, then the message that induces it must be coming from intervals of the form $[a_h, a'_h]$ and $[a_l, a'_l]$ where

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[*] This note contains proofs and discussions in the paper that we claim to be "available in the online supplement." The order follows approximately that of the Appendix of the paper.

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[1] We allow one of the intervals to be empty, that is, $a^j > \tilde{a}^j$ for either $j = h$ or $j = l$, but not both. When $a^h = \tilde{a}^h$ and $a^l = \tilde{a}^l$ both hold, $y(a^h, \tilde{a}^h, a^l, \tilde{a}^l)$ solves $\max_y pU(y - a^h) + (1-p)U(y - a^l)$.

$a_l = a_h + d$ and $a'_l = a'_h + d$. Hence, we study the problem

$$y(a, a', p, d) \equiv \operatorname{argmax}_y V(y, a, a', p, d) \quad \equiv \quad p \int_a^{a'} U(y-s)\, ds + (1-p) \int_{a+d}^{a'+d} U(y-s)\, ds,$$

where $0 \leq a < a' \leq 1$. When $a = a'$, $y(a, a', p, d)$ is simply defined as the y that maximizes $pU(y-a) + (1-p)U(y-(a+d))$.

By virtue of (S-1),[2] we may focus on problems of the form

$$\begin{aligned} y(l, p, d) &\equiv y(0, l, p, d) \\ \equiv \operatorname{argmax}_y V(y, l, p, d) &\equiv p \int_0^{l} U(y-s)\, ds + (1-p) \int_{d}^{l+d} U(y-s)\, ds. \end{aligned} \tag{S-2}$$

We use $l > 0$ to denote the length of the two intervals in which the message is sent. When $l = 0$, the maximand becomes $pU(y) + (1-p)U(y-d)$. Equation (S-1) implies

$$y(a, a', p, d) \quad = \quad y(a' - a, p, d) + a. \tag{S-3}$$

Now we find the solution to Problem (S-2). First, since U is a strictly concave C^2 function,we have $\frac{\partial^2 V}{\partial y^2} < 0$. Therefore,[3] the optimal solution $y(l, p, d)$ is the unique y that satisfies

$$0 = \frac{\partial V}{\partial y} \quad = \quad p \int_0^{l} U'(y-s)\, ds + (1-p) \int_{d}^{l+d} U'(y-s)\, ds, \tag{S-4}$$

if a solution to it exists. Lemma 15 assures its existence.

Proof of Lemma 15, Page 37. First, we show that for $p \in [0, 1]$, $y(l, p, d)$ exists and is in $[\frac{l}{2}, \frac{l}{2} + d]$. It is directly implied by the following three statements: (i) $\frac{\partial V}{\partial y} \geq 0$ for $y \leq \frac{l}{2}$, (ii) $\frac{\partial V}{\partial y} \leq 0$ for $y \geq \frac{l}{2} + d$, and (iii) function $\frac{\partial V}{\partial y}$ is continuous.

Now, we show these three claims. That $\frac{\partial V}{\partial y}$ is continuous is directly implied by U being a C^2 (hence C^1) function. To show the first two claims, it is useful to observe that the symmetry of U implies

$$\int_{-a}^{a} U'(t)\, dt \quad = \quad 0,$$

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[2] We have slightly abused notations with the use of functions y and V, in that their arguments change according to the optimization problem. However, we believe the meaning is clear from the context. In the rest of the Appendix, occurrences of the function y will be as defined in (S-2), unless otherwise noted.

[3] Strictly speaking, we should have made this argument before introducing the normalization based on (S-1). However, it is clear the second order condition can be easily adapted to the problem before the normalization.

which further implies for $a > b \geq 0$,

$$\begin{aligned}\int_{-a}^{b} U'(t)\ dt &= \textstyle\int_{-a}^{-b} U'(t)\ dt > & 0,\\ \int_{-b}^{a} U'(t)\ dt &= \textstyle\int_{b}^{a} U'(t)\ dt < & 0.\end{aligned}$$

Using the above facts, for $y \leq \frac{l}{2}$,

$$\begin{aligned}\frac{\partial V}{\partial y} &= p\int_0^l U'(y-s)\ ds + (1-p)\int_d^{l+d} U'(y-s)\ ds\\ &= \underbrace{p\int_0^{2y} U'(y-s)\ ds}_{=0} + \underbrace{p\int_{2y}^{l} U'(y-s)\ ds}_{\leq 0 \text{ and } =0 \text{ only if } p=0 \text{ or } y=\frac{l}{2}} + \underbrace{(1-p)\int_d^{l+d} U'(y-s)\ ds}_{\leq 0 \text{ and } =0 \text{ only if } p=1 \text{ or } (d=0 \text{ and } y=\frac{l}{2})}\\ &\geq 0.\end{aligned}$$

Note that when $d = 0$, $p = 0$, or $p = 1$, we are back to the Crawford and Sobel model in which there is no uncertainty about bias. Thus, the above inequality holds as strict inequality as long as $p \in (0, 1)$. Similarly, we can prove the second claim that $\frac{\partial V}{\partial y} \leq 0$ for $y \geq \frac{l}{2} + d$ with strict inequality as long as $p \in (0, 1)$. We can then conclude

$$y(l, p, d) \in (\frac{l}{2}, \frac{l}{2} + d) \quad \text{for } p \in (0, 1).$$

The above is also true for the case $l = 0$. In addition, by the Maximum Theorem,[4] the function y is continuous in l, p, and d. Note that it is continuous at $l = 0$ since as $l \to 0$ the first order condition (S-4) becomes equivalent to

$$pU'(y) + (1-p)U'(y-d) = 0,$$

the first order condition that determines $y(0, d, p)$.

In the rest of the proof, we will only deal with the case $p > \frac{1}{2}$. The other two cases will be evident from the proof below. As we have shown above,

$$y(l, p, d) \in (\frac{l}{2}, \frac{l}{2} + d).$$

In order to prove the Lemma, it suffices to show that

$$\frac{\partial V}{\partial y} < 0$$

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[4] See Sundaram, p. 237, Theorem 9.17, part 2.

for $y \in [\frac{l}{2} + \frac{d}{2}, \frac{l}{2} + d)$. Again,

$$\frac{\partial V}{\partial y} = p \int_0^l U'(y-s)\, ds + (1-p) \int_d^{l+d} U'(y-s)\, ds.$$

There are two cases: 1. $l \le d$; 2. $l \ge d$.

1. $l \le d$. We may rewrite the expression for $\frac{\partial V}{\partial y}$ as

   $$\frac{\partial V}{\partial y} = \int_0^l pU'(y-s) + (1-p)U'(y-(l+d-s))\, ds.$$

   When $y \in [\frac{l}{2} + \frac{d}{2}, \frac{l}{2} + d)$, $y \ge l$ since $l \le d$. For all $s \in [0, l]$, we have $y - s \ge y - (l+d-s)$ since $s \le l \le \frac{l}{2} + \frac{d}{2}$ and $y - s \ge (l+d-s) - y$ since $y \ge \frac{l}{2} + \frac{d}{2}$. Therefore, for each $s \in [0, l]$,

   $$y - s \ge |y - (l+d-s)|.$$

   Hence

   $$\begin{aligned} & pU'(y-s) + (1-p)U'(y-(l+d-s)) \\ \le\ & pU'(|y-(l+d-s)|) + (1-p)U'(y-(l+d-s)). \end{aligned}$$

   When $y > l+d-s$, the expression on the right hand side becomes

   $$U'(|y-(l+d-s)|) < 0.$$

   When $y \le l+d-s$, by the symmetry of U, the expression becomes

   $$(2p-1)U'(|y-(l+d-s)|) \le 0$$

   since $p > \frac{1}{2}$, with equality only if $y = l+d-s$. Thus, $\frac{\partial V}{\partial y} < 0$.

2. $l \ge d$. There are two possibilities: $y \ge l$ or $y \le l$.

   2-i. When $y \ge l$, we may rewrite the expression for $\frac{\partial V}{\partial y}$ as

   $$\begin{aligned} \frac{\partial V}{\partial y} = & \int_0^{l-(y-d)} pU'(y-s) + (1-p)U'(y-(l+d-s))\, ds \\ & + p \int_{l-(y-d)}^{l} U'(y-s)\, ds + (1-p) \int_d^y U'(y-s)\, ds. \end{aligned}$$

   The first term on the right hand side is negative by similar reasoning to that of case 1 above. The other two terms are both nonpositive as $y \ge s$ for $s \in [l, l-(y-d)]$ and $s \in [d, y]$, and $U'(x) < 0$ for $x > 0$. Hence, $\frac{\partial V}{\partial y} < 0$.

2-ii. When $y \leq l$, first observe $y \geq \frac{l}{2} + \frac{d}{2}$ implies $y - d \geq l - y$. Now, we may write

$$\begin{aligned} \frac{\partial V}{\partial y} &= p \left[\int_0^{2y-l} U'(y-s) ds + \int_{2y-l}^{l} U'(y-s)\ ds \right] \\ &\quad + (1-p) \left[\int_d^{2y-d} U'(y-s)\ ds + \int_{2y-d}^{l+d} U'(y-s)\ ds \right]. \end{aligned}$$

Note that $\int_{2y-l}^{l} U'(y-s)\ ds = \int_d^{2y-d} U'(y-s)\ ds = 0$ due to symmetry of U. We have

$$\begin{aligned} \frac{\partial V}{\partial y} &= \int_0^{(l+d)-(2y-d)} pU'(y-s) + (1-p)U'(y-(l+d-s))\ ds \\ &\quad + p \int_{(l+d)-(y-d)}^{2y-l} U'(y-s)\ ds. \end{aligned}$$

Again, by the same reasoning as previously used, the first term on the right hand side is negative, and the second term is nonpositive. Hence, $\frac{\partial V}{\partial y} < 0$. $\square$

Proof of Lemma 2, Page 15. Suppose to the contrary, there exist an infinite number of possible actions taken in equilibrium. Denote the set of equilibrium actions by Y^*. By the Weierstrass-Bolzano Theorem, it must have a limit point $y \in [0, 1]$, since it is an infinite subset of a compact set, $[0, 1]$.

First, we argue that y is in neither $[0, b_h)$ nor $(1 + b_l, 1]$ (the latter applies to the case $b_l < 0$). We consider the case $y \in [0, b_h)$ only, and the case $y \in (1 + b_l, 1]$ (when $b_l < 0$) is similar. Suppose $y \in [0, b_h)$. Since y is a limit point of Y^*, the collection of actions in Y^* that are less than b_h, $\{y_\alpha\}$, is infinite. Since the high type's most preferred action in any state, $s + b_h$, is always at least b_h, by Lemma 1 she prefers the higher of any two actions in $\{y_\alpha\}$ in all $s \in [0, 1]$. Therefore, either the high type induces only the maximal action of y_α, or no actions from y_α at all. This implies that there are an infinite number of actions below b_h that are induced only by the low type. But since $b_l \neq 0$, this is impossible by Lemma 1 of Crawford and Sobel (1982).

Thus, we conclude $y \in [b_h, 1 + b_l]$ when $b_l < 0$, and $y \in [b_h, 1]$ when $b_l > 0$.[5] Assume for any $\varepsilon > 0$, there exist an infinite number of actions in Y^*, which are *greater* than y and within ε away from y. Similar arguments can be used for the case where there are an infinite number of actions below and close to y. Now, let

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[5] As a side note, $y \in Y^*$, hence Y^* is closed. Otherwise, in state $y - b_h$ the high type would have no optimal action to induce since there are actions arbitrarily close to y.

$y < y_1 < y_2$ where $y_2 - y < \varepsilon$. By Lemma 1, an expert of bias β strictly prefers y to y_1 to y_2 if $s < \frac{y+y_1}{2} - \beta$, where $\beta = b_h$ or b_l. Similarly, he prefers y_1 to y and y_2 if $s \in (\frac{y+y_1}{2} - \beta, \frac{y+y_2}{2} - \beta)$, and y_2 to y_1 to y if $s > \frac{y+y_2}{2} - \beta$. Let us call $\frac{y+y_1}{2} - \beta$ and $\frac{y+y_2}{2} - \beta$ the cutoff points. As $\varepsilon \to 0$, $y_2 \to y$. Hence the two cutoff points both converge to $y - \beta$. Furthermore, by Lemma 1, since y_1 is neither the smallest nor the largest action in equilibrium, the states in which the high type and low type induce the action y_1 are either two equal-length intervals or two points. Therefore, y_1 has to satisfy the first order condition (S-4). But as the boundary points converge, by the continuity of the integrands in (S-4), the first order condition converges to $pU'(-b_h) + (1-p)U'(-b_l) = 0$, which by symmetry of U implies

$$pU'(b_h) + (1-p)U'(b_l) = 0,$$

contradicting our assumption. Note that the above argument also shows that there cannot be an infinite number of actions induced by both types unless $pU'(b_h) + (1-p)U'(b_l) = 0$, since any limit point of such actions must be in $[b_h, 1+b_l]$.

To conclude, when neither b_l nor $pU'(b_h) + (1-p)U'(b_l)$ is equal to zero, there can only be a finite number of actions in equilibrium. □

Proof of Lemma 16, Page 38.

1. We consider the case $a \geq 0$ only in all proofs. The case $a \leq -d$ is analogous.

   Note that by definition, when $a \geq 0$, $\delta(a, p, b_h, d)$ is the δ that maximizes

   $$p \int_0^a U(a + b_h - \delta - s)\, ds + (1-p) \int_0^{a+d} U(a + b_h - \delta - s)\, ds.$$

   By the strict concavity and C^2 property of U, the above expression is strictly concave in δ. Thus the optimal δ is such that the first order condition holds. That is,

   $$G(\delta, a, p) \equiv -p \int_0^a U'(a + b_h - \delta - s)\, ds - (1-p) \int_0^{a+d} U'(a + b_h - \delta - s)\, ds = 0.$$

   We can rewrite it as

   $$G(\delta, a, p) \equiv -p \int_{b_h - \delta}^{a + b_h - \delta} U'(t)\, dt - (1-p) \int_{b_l - \delta}^{a + b_h - \delta} U'(t)\, dt = 0.$$

   By the implicit function theorem, we have

   $$\frac{\partial \delta}{\partial a} = -\frac{\partial G / \partial a}{\partial G / \partial \delta}.$$

We have

$$\frac{\partial G}{\partial \delta} = p\int_0^a U''(a+b-\delta-s)\ ds + (1-p)\int_0^{a+2b} U''(a+b-\delta-s)\ ds < 0,$$

since U is strictly concave. On the other hand,

$$\frac{\partial G}{\partial a} = -(p+(1-p))U'(a+b_h-\delta) > 0,$$

since the optimal action $y_1(a,p,d) = a+b_h-\delta(a,p,b_h,d)$ must be positive (hence $U'(a+b_h-\delta)<0$) given that $a \geq 0$. Therefore, we conclude

$$\frac{\partial \delta}{\partial a} \ > \ 0.$$

Now, we look at

$$\frac{\partial \delta}{\partial p} \ = \ -\frac{\partial G/\partial p}{\partial G/\partial \delta}.$$

Observe that

$$\frac{\partial G}{\partial p} = -\int_{b_h-\delta}^{a+b_h-\delta} U'(t)\ dt + \int_{b_l-\delta}^{a+b_h-\delta} U'(t)\ dt = \int_{b_l-\delta}^{b_h-\delta} U'(t)dt.$$

When $a=0$,

$$\delta(0,p,b_h,d) = 0 + b_h - \frac{d}{2} = \frac{b_h+b_l}{2} \equiv v.$$

Since we have shown $\frac{\partial \delta}{\partial a} > 0$, we have $\delta(a,p,d) > \frac{b_h+b_l}{2}$ for $a>0$. By the symmetry of U and the fact that $U'(t)>0$ for $t<0$,

$$\int_{b_l-\delta}^{b_h-\delta} U'(t)dt \ \geq \ 0,$$

and $=0$ only when $a=0$. Therefore,

$$\frac{\partial \delta}{\partial p} \ \geq \ 0,$$

and $=0$ only when $a=0$.

2. We only consider the case $l>0$, and the proof can be easily adjusted to accommodate the case $l=0$. By definition, $y(l,p,d)$ is the y that maximizes

$$p\int_0^l U(y-s)\ ds + (1-p)\int_d^{l+d} U(y-s)\ ds.$$

That $y(l,p,d) \in (\frac{l}{2}, \frac{l}{2}+\frac{d}{2})$ has been demonstrated in Lemma 15. Again, since U is strictly concave and C^2, the above expression is strictly concave in y. Thus, $y(l,p,d)$ is the unique y that satisfies the first order condition

$$H(y,l,p) \equiv p\int_0^l U'(y-s)\ ds + (1-p)\int_d^{l+d} U'(y-s)\ ds = 0.$$

Let us call it y^*. By the implicit function theorem, we have

$$\frac{\partial y}{\partial l} = -\frac{\partial H/\partial l}{\partial H/\partial y}.$$

We have

$$\frac{\partial H}{\partial y} = p\int_0^l U''(y^*-s)\ ds + (1-p)\int_d^{l+d} U''(y^*-s)\ ds < 0,$$

since U is strictly concave. On the other hand,

$$\frac{\partial H}{\partial l} = pU'(y^*-l) + (1-p)U'(y^*-(l+d)).$$

Note the first order condition can be rewritten

$$\int_0^l pU'(y^*-s) + (1-p)U'(y^*-(s+d))\ ds = 0.$$

By the strict concavity of U, the integrand above is a strictly increasing function of s. Thus, we have

$$pU'(y^*-0) + (1-p)U'(y^*-(0+d)) < 0 < pU'(y^*-l) + (1-p)U'(y^*-(l+d)).$$

Thus, for $l > 0$,

$$\begin{aligned} \frac{\partial y}{\partial l} &= -\frac{pU'(y^*-l) + (1-p)U'(y^*-(l+d))}{[pU'(y^*-0) + (1-p)U'(y^*-(0+d)] - [pU'(y^*-l) + (1-p)U'(y^*-(l+d))]} \\ &\in (0,1). \end{aligned}$$

Similarly,

$$\frac{\partial y}{\partial p} = -\frac{\partial H/\partial p}{\partial H/\partial y}.$$

Observe that

$$\frac{\partial H}{\partial p} = \int_0^l U'(y^*-s) - U'(y^*-(s+d))\ ds < 0,$$

since U is strictly concave. Thus,

$$\frac{\partial y}{\partial p} < 0.$$

Finally,

$$\frac{\partial y}{\partial d} = -\frac{\partial H/\partial d}{\partial H/\partial y},$$

and

$$\frac{\partial H}{\partial d} = (1-p)U'(y^* - (l+d)) - U'(y^* - d)\ ds \geq 0$$

by the strict concavity of U, with "=" only when $l = 0$. Thus,

$$\frac{\partial y}{\partial d} \geq 0,$$

and $= 0$ only if $l = 0$. □

Supplement to the Proof of Lemma 8, Page 24. In this supplement, we show that A_1 and A_2 as defined in (16) are both less than or equal to zero.

First, we show $A_1 \leq 0$. Note $a_i^h = a_i$ and $a_i^l = a_i + d$ for $i = 1, \ldots, n-1$. Therefore,

$$\begin{aligned} A_1 &= \sum_{i=1}^{n-1} \frac{\partial a_i}{\partial p} \left[p(U(y_i - a_i) - U(y_{i+1} - a_i)) + (1-p)(U(y_i - a_i - d) - U(y_{i+1} - a_i - d)) \right] \\ &= -\sum_{i=1}^{n-2} \frac{\partial a_i}{\partial p} \int_{y_i}^{y_{i+1}} pU'(y - a_i) + (1-p)U'(y - (a_i + d))\ dy. \end{aligned}$$

Note that for each $i = 1, ..., n-1$,

$$\int_{y_i}^{y_{i+1}} pU'(y - a_i) + (1-p)U'(y - (a_i + d))\ dy = \int_{y_i - a_i}^{y_{i+1} - a_i} pU'(t) + (1-p)U'(t-d)\ dt.$$

On the other hand, when $i \neq n-1$, y_{i+1} is the y that satisfies the first order condition

$$\int_{a_i}^{a_{i+1}} pU'(y - s) + (1-p)U'(y - (s+d))\ ds = \int_{y_{i+1} - a_{i+1}}^{y_{i+1} - a_i} pU'(t) + (1-p)U'(t-d)\ dt = 0.$$

We claim

$$y_{i+1} - a_{i+1} \leq y_i - a_i. \tag{S-5}$$

For the $p \geq \frac{1}{2}$ case,[6]

$$\begin{aligned} y_{i+1} - y_i &= a_i + y(a_{i+1} - a_i, p, d) - a_{i-1} - y(a_i - a_{i-1}, p, d) \\ &= 2y(a_{i+1} - a_i, p, d) - 2b_h && \text{(by (11))} \\ &\leq 2(\frac{a_{i+1} - a_i}{2} + \frac{d}{2}) - 2b_h && \text{(since } y(l,p,d) \leq \frac{l}{2} + \frac{d}{2}\text{)} \\ &\leq a_{i+1} - a_i. && \text{(since } d \leq 2b_h\text{)} \end{aligned}$$

Note that equality holds if and only if $p = \frac{1}{2}$ and $b_h = -b_l$. For the case of $b_l \geq 0$, we have $d \leq b_h$. The last two steps above become

$$\begin{aligned} y_{i+1} - y_i &\leq 2(\frac{a_{i+1} - a_i}{2} + d) - 2b_h, && \text{(since } y(l,p,d) \leq \frac{l}{2} + d\text{)} \\ &\leq a_{i+1} - a_i. && \text{(since } d \leq b_h\text{)} \end{aligned}$$

The integrand, $pU'(t) + (1-p)U'(t-d)$, is decreasing in t due to the strict concavity of U. Therefore, for all $i \neq n-1$,

$$\int_{y_i - a_i}^{y_{i+1} - a_i} pU'(t) + (1-p)U'(t-d)\, dt \leq 0.$$

When $i = n-1$,[7] observe that

$$\begin{aligned} (y_{n-1} - a_{n-1}) + (y_n - a_{n-1}) &= 2b_h > 0, \\ (y_{n-1} - a_{n-1}) + (y_n - (a_{n-1} + d)) = (y_n - a_{n-1}) + (y_{n-1} - (a_{n-1} + d)) &= 2b_h - d \geq 0. \end{aligned}$$

By the distance aversion (hence symmetry) and strict concavity of U, we conclude

$$\begin{aligned} U(y_{n-1} - a_{n-1}) - U(y_n - a_{n-1}) &> 0, \\ U(y_{n-1} - a_{n-1}) - U(y_n - a_{n-1}) &\geq \left| U(y_{n-1} - (a_{n-1} + d)) - U(y_n - (a_{n-1} + d)) \right|. \end{aligned}$$

See Figure 1 for a graphical demonstration of these facts.

Also, note that when $b_l \geq 0$,

$$\begin{aligned} & (y_{n-1} - (a_{n-1} + d)) + (y_n - (a_{n-1} + d)) = 2b_h - 2d = 2b_l && \geq 0, \\ \Rightarrow \quad & U(y_{n-1} - (a_{n-1} + d)) - U(y_n - (a_{n-1} + d)) && \geq 0. \end{aligned}$$

Thus, when $p \geq \frac{1}{2}$ or $b_l \geq 0$, we conclude

$$p[U(y_{n-1} - a_{n-1}) - U(y_n - a_{n-1})] + (1-p)[U(y_{n-1} - (a_{n-1} + d)) - U(y_n - (a_{n-1} + d))] \geq 0,$$

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[6] Although the derivation assumes $i \geq 2$, we may use (11) to get the same conclusion for $i = 1$. The only difference lies in the first step of the argument.

[7] In fact, the argument works for all $i = 1, ..., n-1$. But the fact $y_i - a_i \leq y_{i+1} - a_{i+1}$ is also useful in proving $A_2 \leq 0$ and Theorem 11.

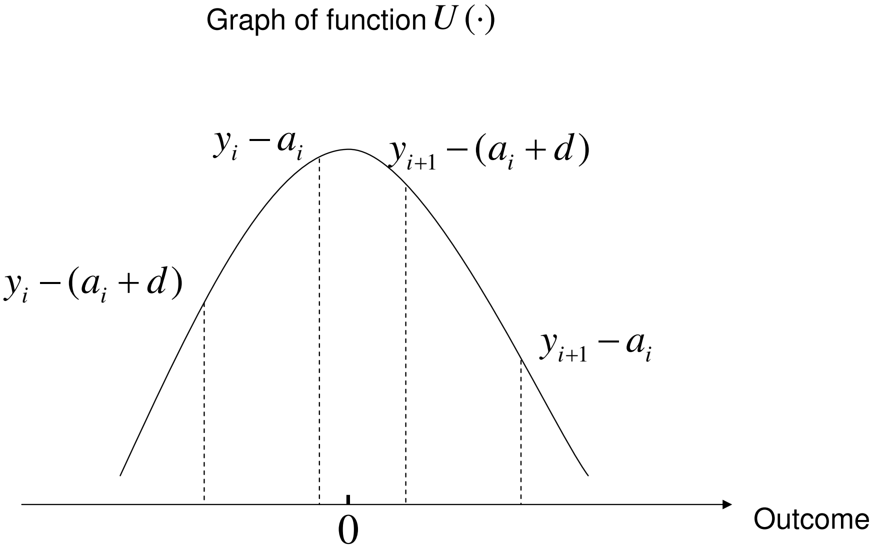

Figure 1: Demonstration of argument for $A_1 \leq 0$. Note that $y_i - a_i$ may be to the right of $y_{i+1} - (a_i + d)$, but the larger of the two must be nonnegative. Also, when $b_l \geq 0$, $y_{i+1} - (a_i + d)$ is positive and farther from 0 than $y_i - (a_i + d)$. Note $i = 1, ..., n-1$.

with equality if and only if $p = \frac{1}{2}$ and $b_h = -b_l$.

Combining the above discussion with the fact $\frac{\partial a_i}{\partial p} < 0$, we conclude

$$A_1 \leq 0,$$

with equality only when $p = \frac{1}{2}$ and $b_h = -b_l$.

Now, we show $A_2 \leq 0$. Note that

$$\begin{aligned} & \textstyle\sum_{i=1}^{n} \int_{a_{i-1}^h}^{a_i^h} U(y_i - s)ds \\ = \quad & \textstyle\int_{y_1 - a_1}^{y_1} U(t)dt + \int_{y_2 - a_2}^{y_2 - a_1} U(t)dt + \ldots + \int_{y_n - 1}^{y_n - a_{n-1}} U(t)dt \end{aligned}$$

For $i = 2, \ldots, n$, using $a_{i-1} = \frac{y_i + y_{i-1}}{2} - b_h$, we have $-(y_i - a_{i-1}) = y_{i-1} - (a_{i-1} + 2b_h)$ and $-(y_i - (a_{i-1} + d)) = y_{i-1} - (a_{i-1} + 2v)$. Symmetry of U implies $\int_x^{x'} U(t)dt = \int_{-x'}^{-x} U(t)dt$. Therefore,

$$\begin{aligned} & \textstyle\sum_{i=1}^{n} \int_{a_{i-1}^h}^{a_i^h} U(y_i - s)ds \\ = \quad & \textstyle\int_{y_1 - a_1}^{y_1} + \sum_{i=2}^{n-1} \int_{y_{i-1} - (a_{i-1} + 2b_h)}^{a_i - y_i} + \int_{y_{n-1} - (a_{n-1} + 2b_h)}^{1 - y_n} . \end{aligned}$$

We have omitted the integrand $U(t)dt$ above, as we will hereafter. Similarly,

$$\begin{aligned} & \textstyle\sum_{i=1}^{n} \int_{a_{i-1}^l}^{a_i^l} U(y_i - s)ds \\ = \quad & \textstyle\int_{y_1 - (a_1 + d)}^{y_1} + \sum_{i=2}^{n-1} \int_{y_{i-1} - (a_{i-1} + 2v)}^{(a_i + d) - y_i} + \int_{y_{n-1} - (a_{n-1} + 2v)}^{1 - y_n} . \end{aligned}$$

Thus,

$$\begin{aligned}
A_2 \;=\;& \int_{y_1-a_1}^{y_1} + \sum_{i=2}^{n-1}\int_{y_{i-1}-(a_{i-1}+2b_h)}^{a_i-y_i} + \int_{y_{n-1}-(a_{n-1}+2b_h)}^{1-y_n} \\
& - \int_{y_1-(a_1+d)}^{y_1} - \sum_{i=2}^{n-1}\int_{y_{i-1}-(a_{i-1}+2v)}^{(a_i+d)-y_i} - \int_{y_{n-1}-(a_{n-1}+2v)}^{1-y_n} \\
=\;& -\int_{y_1-(a_1+d)}^{y_1-a_1} + \int_{y_1-(a_1+2b_h)}^{y_1-(a_1+2v)} - \int_{a_2-y_2}^{(a_2+d)-y_2} + \ldots + \int_{y_{n-1}-(a_{n-1}+2b_h)}^{y_{n-1}-(a_{n-1}+2v)}
\end{aligned}$$

$$\begin{aligned}
=\;& \sum_{i=1}^{n-2}\left[-\int_{y_i-(a_i+2v)}^{y_i-a_i} + \int_{y_i-(a_i+2b_h)}^{y_i-(a_i+d)}\right] - \int_{y_{n-1}-(a_{n-1}+d)}^{y_{n-1}-a_{n-1}} + \int_{y_{n-1}-(a_{n-1}+2b_h)}^{y_{n-1}-(a_{n-1}+2v)} \quad \left(\text{by } \int_x^{x'} = \int_{-x'}^{-x}\right) \\
=\;& \sum_{i=1}^{n-1}\int_{y_i-(a_i+2v)}^{y_i-a_i} U(t-d) - U(t)\;dt.
\end{aligned}$$

By the definition of $\delta(\cdot)$ and (10),

$$y_1 - a_1 = -\delta(a_1, p, b_h, d) + b_h \leq -v + b_h = \frac{d}{2}.$$

As we have shown above, whenever $p \geq \frac{1}{2}$ or $b_l \geq 0$, we have $y_{i+1} - a_{i+1} \leq y_i - a_i$ for all $i = 1, \ldots, n-2$. Thus, for all $i = 1, \ldots, n-1$, any $t \in [y_i - (a_i + 2v), y_i - a_i]$ satisfies

$$|t - d| = d - t \geq |t| \Rightarrow U(t-d) \leq U(t).$$

Therefore,

$$A_2 \leq 0,$$

with strict inequality unless $v = 0$, i.e., $b_h = -b_l$. □

Proof of Lemma 17, Page 46. Let $a_0 = 0$ and $a_i = \sum_{j=1}^{i} l_j$ for $i = 1, ..., n$ (the notation facilitates the application of the lemma to the proof of the theorem), and a_i' be defined similarly. Then, $l_i = a_i - a_{i-1}$.

Let $l_i^\lambda = \lambda l_i + (1-\lambda)l_i'$. Note that $l_i^1 = l_i$ and $l_i^0 = l_i'$. Then, $\lambda_{i+1}^\lambda \geq l_i^\lambda$ for all $i = 1, ..., n-1$. Define

$$V(\lambda) \;=\; \sum_{i=1}^{n}\int_0^{l_i^\lambda} f\left(\frac{l_i^\lambda}{2} - s\right)ds = \sum_{i=1}^{n}\int_{-\frac{l_i^\lambda}{2}}^{\frac{l_i^\lambda}{2}} f(t)dt.$$

To show the lemma, it is sufficient to show that $V(\lambda)$ is increasing in λ. Observe

$$\frac{dV(\lambda)}{d\lambda} \;=\; \sum_{i=1}^{n}\frac{1}{2}(l_i - l_i')\left[f\left(\frac{l_i^\lambda}{2}\right) + f\left(-\frac{l_i^\lambda}{2}\right)\right].$$

Since f is concave and $l_{i+1}^{\lambda} \geq l_i^{\lambda} \geq 0$, we have $f(\frac{l_{i+1}^{\lambda}}{2}) - f(\frac{l_i^{\lambda}}{2}) \leq f(-\frac{l_i^{\lambda}}{2}) - f(-\frac{l_{i+1}^{\lambda}}{2})$, which in turn implies

$$f(\frac{l_{i+1}^{\lambda}}{2}) + f(-\frac{l_{i+1}^{\lambda}}{2}) \leq f(-\frac{l_i^{\lambda}}{2}) + f(\frac{l_i^{\lambda}}{2}).$$

Observe that $l_i - l_i'$ decreases as i increases. Let $m = \max\{i \,|\, l_i \geq l_i'\}$. Thus, $l_i \geq l_i'$ for all $i \leq m$ and $l_i < l_i'$ for all $i > m$ (if $m < n$). Note also the only case in which $m = n$ is that when $l_i = l_i'$ for all i, in which case $dV(\lambda)/d\lambda = 0$. When $l_i \neq l_i'$ for some i,

$$\begin{aligned}
\frac{dV(\lambda)}{d\lambda} &\geq \sum_{i=1}^{m} \frac{1}{2}(l_i - l_i')[f(\frac{l_m^{\lambda}}{2}) + f(-\frac{l_m^{\lambda}}{2})] + \sum_{i=m+1}^{n} \frac{1}{2}(l_i - l_i')[f(\frac{l_{m+1}^{\lambda}}{2}) + f(-\frac{l_{m+1}^{\lambda}}{2})] \\
&= \frac{1}{2}(a_m - a_m') \left[[f(\frac{l_m^{\lambda}}{2}) + f(-\frac{l_m^{\lambda}}{2})] - [f(\frac{l_{m+1}^{\lambda}}{2}) + f(-\frac{l_{m+1}^{\lambda}}{2})] \right] \\
&\geq 0.
\end{aligned}$$

Also, the inequality is strict if f is strictly concave unless $\lambda = 0$ or 1. $\square$

Supplement to the Proof of Theorem 10, Page 27. Here, we demonstrate that when $b_h = -b_l = b$ and $p \geq \frac{1}{2}$, in any $n-$action nondisclosure equilibrium ($n \geq 2$), the low type's expected utility is higher than the high type's. Let us denote type j's ($j = h, l$) payoff in the equilibrium by V_j. Thus,

$$V_h - V_l = \sum_{i=1}^{n} \int_{a_{i-1}^h}^{a_i^h} \tilde{U}(y_i - (s+b))ds - \sum_{i=1}^{n} \int_{a_{i-1}^l}^{a_i^l} \tilde{U}(y_i - (s-b))ds.$$

Recall that $a_0^j = 0$ and $a_n^j = 1$ for $j = h, l$ and that $a_i^h = a_i$ and $a_i^l = a_i + d = a_i + 2b$ for $i = 2, \ldots, n-1$. Substituting them into the above equation, we have

$$\begin{aligned}
& V_h - V_l \\
= & \left[\textstyle\int_0^{a_1} \tilde{U}(y_1 - (s+b))ds + \int_{a_{n-1}}^{1} \tilde{U}(y_n - (s+b))ds \right] \\
& - \left[\textstyle\int_0^{a_1+2b} \tilde{U}(y_1 - (s-b))ds + \int_{a_{n-1}+2b}^{1} \tilde{U}(y_n - (s-b))ds \right] \\
= & - \textstyle\int_0^{2b} \tilde{U}(y_1 - (s-b))ds + \int_{1-2b}^{1} \tilde{U}(y_n - (s+b))ds \\
= & - \textstyle\int_{-b}^{b} \tilde{U}(y_1 - t)dt + \int_{-b}^{b} \tilde{U}(y_n - 1 - t)dt.
\end{aligned}$$

Clearly, $y_1 > 0$ and $y_n - 1 < 0$. By the distance-aversion property of $\tilde{U}$, $V_h - V_l \leq 0$ if and only if $y_1 + y_n - 1 \leq 0$. The optimality of the decision maker's strategy gives us

$$\begin{aligned}
p \int_0^{a_1} U'(y_1 - s)ds + (1-p) \int_0^{a_1+2b} U'(y_1 - s)ds &= 0, \\
p \int_{a_{n-1}}^{1} U'(y_n - s)ds + (1-p) \int_{a_{n-1}+2b}^{1} U'(y_n - s)ds &= 0.
\end{aligned}$$

By symmetry of the decision maker's and the expert's preferences, $1 - a_{n-1} = a_1 + 2b$ when $p = \frac{1}{2}$. From Corollary 6, for all $p \geq \frac{1}{2}$, we have $1 - a_{n-1} \geq a_1 + 2b$. Let us replace y_n with $1 - y_1$ in the left hand side of the second equation above. If the expression is nonpositive, then due to concavity of U, we may conclude $1 - y_1 \geq y_n$, or $y_1 + y_n - 1 \leq 0$. But,

$$
\begin{aligned}
& p \int_{a_{n-1}}^{1} U'(1 - y_1 - s)ds + (1 - p) \int_{a_{n-1}+2b}^{1} U'(1 - y_1 - s)ds \\
= \; & -\left[p \int_{0}^{1-a_{n-1}} U'(y_1 - t)dt + (1 - p) \int_{0}^{1-a_{n-1}-2b} U'(y_1 - t)dt\right] \\
= \; & -\left[p \int_{a_1}^{1-a_{n-1}} U'(y_1 - t)dt - (1 - p) \int_{1-a_{n-1}-2b}^{a_1+2b} U'(y_1 - t)dt\right] \\
= \; & -\left[(2p - 1) \int_{a_1}^{a_1+2b} U'(y_1 - t)dt + p \int_{a_1+2b}^{1-a_{n-1}} U'(y_1 - t)dt + (1 - p) \int_{a_1}^{1-a_{n-1}-2b} U'(y_1 - t)dt\right].
\end{aligned}
$$

The second equality sign above comes from the first order condition for y_1. Using the facts $y_1 \leq a_1 + b$, $p \geq \frac{1}{2}$, $U'(x) > 0$ for $x < 0$, and the symmetry and concavity of U, we conclude the above expression is nonpositive. Hence the desired result $V_h - V_l \leq 0$. $\square$