Proof of Lemma 1. In any pure strategy equilibrium, symmetry implies $m_0^i(0) = 0$. To show the other statement, it is enough to consider $m_0^A(1)$ and $m_1^A(1)$.
First $m_1^A(1) = 1$ by informativeness and monotonicity. For an expert of type 0, it is always true that $u(y(s, m_x^B(1)), 1, 0) \geq u(y(t, m_x^B(1)), 1, 0)$ for all $s, t \in S, s \geq t$ and $x \in X$. Thus
(*) $\frac{1}{3}\sum_{x\in X} u(y(s, m_x^B(1)), 1, 0) \geq \frac{1}{3}\sum_{x\in X} u(y(t, m_x^B(1)), 1, 0)$, for all $s, t \in S, s \geq t$,
making $m_0^A(1) = 1$ a best response no matter what the other does. Now we check that this is the only possible strategy in equilibrium.
First $m_0^A(1) = -1$ would violate monotonicity.
Suppose $m_0^A(1) = 0$. Due to symmetry and anonymity, $y(1, -1) = y(-1, 1) = -y(1, -1)$. Therefore, they must both be 0. Informativeness gives us $y(1, 1) > 0 = y(-1, 1)$, which in turn implies that either $y(1, 1) > y(0, 1)$ or $y(1, 0) > y(0, 0)$ must be true. Thus (*) holds strictly for $s = 1$ and $t = 0$, since $m_1^B(1) = 1$ and $m_0^B(1) = 0$. This contradicts $m_0^A(1) = 0$ being a best response.
The above arguments prove the lemma. Note the arguments do not rely on the magnitude of b. □

Proof of Proposition 2. Given Lemma 1, the only strategies left to be determined are $m_s^i(-s)$ and $m_s^i(0)$. Once they are determined, y can be derived from (EQ3). By symmetry and anonymity, it is enough to discuss $m_1^A(-1)$ and $m_1^A(0)$.
First I consider $m_1^A(0)$.

(a) $m_1^A(0) = -1$. This would violate monotonicity.

(b) $m_1^A(0) = 0$. By the following argument, this cannot be part of equilibrium either.
By symmetry $m_{-1}^A(0) = 0$. Thus $\frac{1}{3}\sum_{x\in X} u(y(0, m_x^A(0)), 0, b_1) = u(y(0, 0), 0, b_1)$.
Thus we have
$\frac{1}{3}\sum_{x\in X} u(y(1, m_x^A(0)), 0, b_1) > \frac{1}{3}\sum_{x\in X} u(y(0, m_x^A(0)), 0, b_1)$,
unless $y(1, 0) = y(0, 0)$. But if $y(1, 0) = y(0, 0) = 0$, then it must be that $m_{-1}^A(1) \neq 0$, otherwise $y(1, 0) = 1$ by Lemma 1. As shown above $y(1, -1) = 0$. But $m_1^A(-1) = 1$ and $m_1^A(-1) = -1$ could not be part of an equilibrium.
When $m_1^A(-1) = 1$, $y(1, m_{-1}^A(-1)) = y(1, m_0^A(-1)) = y(1, -1) = y(0, -1)$=0, while $y(1, m_1^A(-1)) = y(1, 1) > y(0, 1) > -1 + b$. Since $-1 + b$ is type 1 expert's most preferred action, $m_1^A(-1) = 0$ is better than $m_1^A(-1) = 1$.
When $m_1^A(-1) = -1$, $y(1, 1) = 1$. Thus $y(-1, m_x^A(-1)) = y(-1, -1) = -1$

for all x and $y(0,-1)=0$, and the latter is closer to a type 1 expert's most preferred action $-1+b$ since $b>\frac{1}{2}$.

(c) $m_1^A(0)=1$. Now, I discuss $m_1^A(-1)$.

(i) $m_1^A(-1)=-1$. Given this, we can calculate the eventual probability distribution over signal pairs and the decision maker's optimal strategy. The following is a list:[1]

(m^A,m^B)	$\text{Prob}(m^A,m^B,s=-1+0+1)$	$y(m^A,m^B)$
$(1,1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}$	$\frac{9}{10}$
$(1,0)$ or $(0,1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$	0
$(1,-1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$	0
$(0,0)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$	0

The probabilities are calculated according to the experts' strategies. For example, $\text{Prob}(0,1,s=0)=P(s=0)\times P(x^A=0)\times P(x^B=1)$, because in state 0, only type 0 experts report 0 and only type 1 experts report 1. The probabilities and decisions for the omitted pairs can be inferred from symmetry and anonymity. Take $(-1,-1)$ for example. The probabilities should be derived from $(1,1)$, which are $\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$, and the decision should be $-y(1,1)=-\frac{9}{10}$.
Given these we may check the optimality of strategies of experts. Now given $s=-1$, an expert of type 1 would earn $u(-\frac{9}{10},-1,b)$ if she chooses $m_1^A(-1)=-1$, or $u(0,-1,b)$ if she chooses $m_1^A(-1)=0$. The latter is higher since 0 is closer to $-1+b$ (her most preferred action when $s=-1$) than $-\frac{9}{10}$ is, as $b\geq\frac{17}{21}>\frac{11}{20}$. So, $m_1^A(-1)=-1$ is not optimal.

(ii) $m_1^A(-1)=0$. Again, the following is the table of probabilities and decisions:

(m^A,m^B)	$\text{Prob}(m^A,m^B,s=-1+0+1)$	$y(m^A,m^B)$
$(1,1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}$	$\frac{4}{5}$
$(1,0)$ or $(0,1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}$	$\frac{2}{3}$
$(1,-1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$	0
$(0,0)$	$\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$	0

[1]The notation $\text{Prob}(m^A,m^B,s=-1+0+1)$ means that in that column, probabilities of (m^A,m^B,s) are separated by "+" according to different s.

Now we check the optimality of $m_1^A(-1) = 0$. I omit the calculation here, but as long as $b > \frac{58}{105}$, it turns out for $t = -1$ and 1
$\frac{1}{3}\sum_{x\in X} u(y(0, m_x^A(0)), -1, b_1) > \frac{1}{3}\sum_{x\in X} u(y(t, m_x^A(0)), -1, b_1)$,
which is certainly satisfied since $b \geq \frac{17}{21} > \frac{58}{105}$.
Thus $m_1^A(-1) = 0$ is optimal. Note that this is exactly **strategy profile (A)**. The expected payoff of the decision maker when $s = 1$ or $s = -1$ is

$$\begin{aligned}
& -[P(x^A = 1 \text{ or } 0, x^B = 1 \text{ or } 0) \times (y(1,1) - 1)^2 \\
& +2 \times P(x^A = -1, x^B = 1 \text{ or } 0) \times (y(0,1) - 1)^2 \\
& +P(x^A = -1, x^B = -1) \times (y(0,0) - 1)^2] \\
= & -[\frac{2}{3} \times \frac{2}{3} \times (\frac{4}{5} - 1)^2 + 2 \times \frac{1}{3} \times \frac{2}{3} \times (\frac{2}{3} - 1)^2 + \frac{1}{3} \times \frac{1}{3} \times (0 - 1)^2] \\
= & -\frac{361}{2025}.
\end{aligned}$$

His expected payoff when $s = 0$ is

$$\begin{aligned}
& -[2 \times P(x^A = 1, x^B = 1) \times (y(1,1) - 0)^2 \\
& +2 \times P(x^A = 0, x^B = 1 \text{ or } -1) \times (y(0,1) - 1)^2] \\
= & -[2 \times \frac{1}{3} \times \frac{1}{3} \times (\frac{4}{5} - 0)^2 + 2 \times \frac{1}{3} \times \frac{2}{3} \times (\frac{2}{3} - 0)^2] \\
= & -\frac{688}{2025}.
\end{aligned}$$

Thus his expected payoff is $-\frac{1}{3}(\frac{361}{2025} + \frac{688}{2025} + \frac{361}{2025}) = -\mathbf{\frac{94}{405}}$.

(iii) $m_1^A(-1) = 1$. The following is the table of probabilities and decisions:

(m^A, m^B)	$\text{Prob}(m^A, m^B, s = -1 + 0 + 1)$	$y(m^A, m^B)$
$(1,1)$	$\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\times\frac{2}{3}$	$\frac{1}{2}$
$(1,0)$ or $(0,1)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$	0
$(1,-1)$	$\frac{1}{3}\times\frac{1}{3}\times\frac{2}{3}+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}$	0
$(0,0)$	$0+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}+0$	0

Now we check the optimality of $m_1^A(-1) = 1$. Note that $u(y(0,1), -1, b_1) = u(0, -1, b_1) > u(1/2, -1, b_1) = u(y(1,1), -1, b_1)$ and $u(y(0,-1), -1, b_1) = u(0, -1, b_1) = u(y(1,-1), -1, b_1)$. Thus $m_1^A(-1) = 0$ is strictly better than $m_1^A(-1) = 1$. Hence $m_1^A(-1) = 1$ is not optimal.

Summarizing the above arguments gives us the proposition. □

Proof of Lemma 2. By symmetry $m_0^A(0) = 0$, $m_0^B(0,0) = 0$ and $y(0,0) = 0$. Due to monotonicity and the rationalizability requirement that $y(m^A, m^B) \in [-1,1]$, I have $m_0^B(1, m^A), m_1^B(1, m^A) \in \arg\max_{m^B} y(m^A, m^B)$. By monotonicity $m_1^A(1) = 1$, otherwise $m_x^A(s) = 0$ for all $x \in X$ and $s \in S$, making the first expert's reports uninformative, violating the informativeness condition.

Now I show Part 3. By monotonicity, $y(1,1) \geq y(1,0) \geq y(0,0) = 0$. But in state 0, the most preferred action for an expert of type -1 is $-b < -\frac{1}{2}$. Thus, $y(1,-1)$ is as good as or better than $y(1,0)$ and $y(1,1)$ for her. Hence, $m_{-1}^B(0,1) \in \arg\min_{m^B} y(1, m^B)$. To show that $m_1^B(0,0) \in \arg\max_{m^B} y(0, m^B)$, note that $y(0,1) \geq y(0,0) = 0 \geq y(0,-1)$. Thus $y(0,1)$ is closer to b than $y(0,0)$ and $y(0,-1)$ since $b > \frac{1}{2}$. Hence $m_1^B(0,0) \in \arg\max_{m^B} y(0, m^B)$.

To show $m_0^A(1) = 1$, it suffices to argue that $m_0^A(1) = 0$ is impossible in equilibrium, as $m_0^A(1) = -1$ would violate monotonicity.

Suppoe $m_0^A(1) = 0$ (hence $m_0^A(-1) = 0$ by symmetry). By monotonicity, $m_1^A(-1) = 0$ or 1.

1. Now consider the case $m_1^A(-1) = 0$.

   (a) $m_1^A(0) = 0$. Then $y(1, m^B) = 1$ for all $(1, m^B)$ that are sent in equilibrium since $m^A = 1$ only when $s = 1$. Given this $m_0^A(1) = 0$ could be optimal only if $y(0,1) = 1$ and $m_{-1}^B(1,0) = 1$. But the latter is not optimal for the expert, since $b > \frac{1}{2}$ implies that $y(0,0)$ is closer to her most preferred action $1 - b$ than $y(0,1) = 1$.

   (b) $m_1^A(0) = 1$. First, consider the case $y(0, m_B) = 0$ for all m_B. Monotonicity implies $y(1,-1) \geq 0$. Thus, $m_0^A(1) = 0$ guarantees an action of 0 from the decision maker, while $m_0^A(1) = 1$ guarantees nonnegative actions from the decision maker and strictly positive action some of the time due to informativeness. Thus, $m_0^A(1) = 0$ is not optimal.

   If $y(0,1) > y(0,0)$ then $m_1^B(-1,0) = 0$ or -1, since in state -1 a type 1 expert's most preferred action is $-1 + b < 0$. By part 2 of this lemma, $m_1^B(1,0) = m_0^B(1,0) = 1$, which implies $y(0,1) \geq 4/5$. This has two implications. The first one is $m_1^B(-1,0) = 0$ since $-1 + b \geq -1 + \frac{17}{21} > -\frac{2}{5}$. This in turn implies $y(0,1) = \frac{4}{5}$. The second one is that by monotonicity $y(1,1) \geq 4/5$. On the other hand, by the determined parts of the strategy profile, $y(1, m^B) \leq \frac{3}{4}$ for any m^B if $m_v^B(0,1) = m^B$ for some $v \in X$. But as proven above $m_0^B(1,1), m_1^B(1,1) \in \arg\max_{m^B} y(1, m^B)$. Thus $y(1,m) \geq \frac{4}{5}$ for all $m \in \arg\max_{m^B} y(1, m^B)$, which leaves the only possibility $\max_{m^B} y(1, m^B) = 1$. Hence $m_0^A(1) = 0$ induces actions $y(0, 1) = \frac{4}{5}$ with probability $\frac{2}{3}$ and 0 with probability $\frac{1}{3}$, while $m_0^A(1) = 1$ induces actions $\max_{m^B} y(1, m^B) = 1$ with probability $\frac{2}{3}$ and another non-negative action with probability $\frac{1}{3}$. This means that $m_0^A(1) = 1$ is a better response.

2. Now we consider the case $m_1^A(-1) = 1$. We get $m_1^A(0) = 1$ by monotonicity. Thus

$$\sum_{m^B} P(m^B|m^A = 0)y(0, m^B) = \sum_{m^B} P(m^B|m^A = 1)y(1, m^B) = 0.$$

Since $m^A = 1$ happens in state -1, by monotonicity $y(1, -1) < 0$, which in turn implies $y(0, -1) < 0$ and $y(0, 1) > 0$. Thus, by parts 2 and 3 (proven above) of this Lemma, I have $m_1^B(1, -1) = m_0^B(1, -1) = 1$ and $m_1^B(0, -1) = 1$. I can also conclude that $m_1^B(-1, 0) = 0$ or -1 since $y(0, 1)$ is positive and hence farther from $-1 + b$ than $y(0, 0) = 0$.

If $m_1^B(-1, 0) = -1$ then by the proven parts of this lemma $y(0, -1) = -3/4$, which makes $m_1^B(-1, 0) = -1$ not optimal since $y(0, 0) = 0$ is closer to $-1 + b \geq -1 + \frac{17}{21} > -\frac{3}{8}$.

So $m_1^B(-1, 0) = 0$ and $y(0, -1) = -2/3$. This implies $y(1, -1) \geq -2/3$ and $y(-1, -1) \leq -2/3$ by monotonicity (by symmetry, $y(-1, 1) \leq 2/3$ and $y(1, 1) \geq 2/3$). These imply $m_1^B(-1, 0) = 0$ is optimal, since a type 1 expert's most preferred action in state -1 is $-1 + b \in (-\frac{1}{3}, 0)$.

As proven above $m_1^B(1, 1), m_0^B(1, 1) \in \arg\max_{m^B} y(1, m^B)$. I separate the discussion into two different cases.

   (a) $y(1, 0) = y(1, 1) \geq \frac{2}{3}$.

   I claim $m_0^B(0, 1) = -1$. Suppose to the contrary $m_0^B(0, 1) = 0$ or 1. I have proved that $m_1^B(0, 1) = 0$ or 1. This would cause $P(s = 1|(m^A, m^B) = (1, 0)$ or $(1, 1)) \leq \frac{2}{5}$, contradicting the assumption $y(1, 0) = y(1, 1) \geq \frac{2}{3}$.

   It is also necessary to have $m_1^B(-1, 1) = -1$, otherwise $y(1, 0) \leq \frac{2}{5}$.

   Now if $m_{-1}^B(1, 1) = 0$ or 1, then $m_0^A(1) = 1$ is a better response than $m_0^A(1) = 0$. Since the former ensures that the decision maker takes the action $y(1, 0) = y(1, 1) \geq \frac{2}{3}$, while the latter induces actions $y(0, 0) = 0$ with probability $\frac{1}{3}$ and $y(0, 1) = \frac{2}{3}$ with probability $\frac{2}{3}$.

   If $m_{-1}^B(1, 1) = -1$ then $y(1, -1) = -\frac{1}{3}$, and $y(1, 0) = y(1, 1) = \frac{2}{3}$. But in this case $m_1^A(1) = 1$ would be a worse response than $m_1^A(1) = 0$, a contradiction. The former induces actions $y(1, -1) = -\frac{1}{3}$ with probability

$\frac{1}{3}$ and $y(1,0) = y(1,1) = \frac{2}{3}$ with probability $\frac{2}{3}$, while the latter induces the same distribution of actions described in the previous paragraph.

(b) $y(1,1) > y(1,0)$. This implies $m_1^B(1,1) = m_0^B(1,1) = 1$, $m_1^B(-1,1) \neq 1$, and $m_0^B(0,1) \neq 1$. We may also conclude that $m_{-1}^B(1,1) \neq 1$ since that would imply $y(1,1) \geq \frac{3}{4}$, $y(1,0) \geq 0$, and a type -1 expert's most preferred action in state 1 is $1 - b \leq \frac{1}{3}$. Now we consider the two possibilities:

i. $m_1^B(0,1) = 0$. This implies that $y(1,1) = 1$. If $m_{-1}^B(1,1) = -1$ then $y(1,0) = 0$ as $y(1,0) \geq y(0,0) = 0$ by monotonicity. But then $m_{-1}^B(1,1) = 0$ is better than $m_{-1}^B(1,1) = -1$ since $y(1,-1) < 0$ and $b < 1$. Thus $m_{-1}^B(1,1) = 0$. Now $m_0^A(1) = 1$ is a better response than $m_0^A(1) = 0$ since the former induces $y(1,0) \geq 0$ with probability $\frac{1}{3}$ and $y(1,1) = 1$ with probability $\frac{2}{3}$, while the latter induces $y(0,0) = 0$ with probability $\frac{1}{3}$ and $y(0,1) = \frac{2}{3}$ with probability $\frac{2}{3}$.

ii. $m_1^B(0,1) = 1$. This implies $y(1,1) = \frac{2}{3}$.
If $m_{-1}^B(1,1) = -1$, I have $m_0^B(0,1) = -1$. The reason is that $m_0^B(0,1) = 0$ implies $y(1,0) = 0$, and $m_{-1}^B(1,1) = 0$ would be a better response than $m_{-1}^B(1,1) = -1$ as her most preferred action is $1 - b > 0$. Thus, $y(1,-1) < 0 = y(0,0)$ and $y(1,1) = y(0,1)$, and $m_1^A(1) = 1$ would not be optimal. A contradiction.
If $m_{-1}^B(1,1) = 0$ then $m_0^A(1) = 1$ is a better response than $m_0^A(1) = 0$ and constitutes a contradiction unless $y(1,0) = 0$. To make $y(1,0) = 0$ we need $m_1^B(-1,1) = 0$, and $y(1,0) = 0$ implies $m_0^B(0,1) = 0$. Thus the first expert's report is completely uninformative, violating informativeness.

The above arguments show that the only possibility is $m_0^A(1) = 1$. □

Proof of Proposition 3. In what follows, I show that strategy profiles (B), (C), and (D) are the only equilibria are the only ones where both experts provide information to the decision maker. I show that (C) gives the decision maker higher payoff than (B) and (D). Therefore, it is the most informative equilibrium.

By Lemma 2, the only parts of expert A's strategy left to be determined are $m_1^A(-1)$ and $m_1^A(0)$. I proceed by considering all combinations allowable under monotonicity and informativeness.

Case 1. $m_1^A(-1) = -1$ and $m_1^A(0) = 0$. No matter what expert B does, the state is perfectly revealed to the decision maker. But given this $m_1^A(-1) = -1$ is not

Table 1: Strategy Profile (B)

m^A	Type 0	Type 1
State -1	-1	0
State 0	0	1
State 1	1	1

m_0^B	$m^A=-1$	$m^A=0$	$m^A=1$
State -1	-1	-1	-1
State 0	1	0	-1
State 1	1	1	1

m_1^B	$m^A=-1$	$m^A=0$	$m^A=1$
State -1	-1	0	1
State 0	1	1	1
State 1	1	1	1

y	$m_B=0$	$m_B=1$
$m_A=-1$	$-2/3$	$-2/3$
$m_A=0$	0	$2/3$
$m_A=1$	$2/3$	$2/3$

optimal since 0 is closer than -1 is to her most preferred action $-1+b > -\frac{1}{2}$. The argument works as long as $b > \frac{1}{2}$.

Case 2. $m_1^A(-1) = -1$ and $m_1^A(0) = 1$. In this case since $m^A = 0$ only happens when $s = 0$, $y(0, m^B) = 0$ for all m^B such that $m_v^B(0,0) = m^B$ for some $v \in X$. Now in order for the second expert's report to be informative, we need $y(1,1) > y(1,-1)$. This implies $m_0^B(0,1) \neq 1$. Note that $y(1,1) \geq \frac{3}{4}$ since $P(s=1|m^A=1) = \frac{3}{4}$ and $s \geq 0$ when $m^A = 1$. Thus $y(-1,-1) \leq -\frac{3}{4}$.
Now compare $m_1^A(-1) = 0$ with $m_1^A(-1) = -1$. The former induces action 0 by the decision maker, while the latter induces $y(-1,-1) \leq -\frac{3}{4}$ with probability $\frac{2}{3}$. The difference in expected utility is thus greater than

$$-(0-(-1+b))^2 - [-\frac{2}{3}(-\frac{3}{4} - (-1+b))^2] = -\frac{1}{3}(b^2 - 5b + \frac{23}{8}) > 0$$

for all $b \geq \frac{17}{21} > \frac{10-3\sqrt{6}}{4}$. Hence $m_1^A(-1) = 0$ is a better response, a contradiction. The argument works as long as $b > \frac{10-3\sqrt{6}}{4}$.

Case 3. $m_1^A(-1) = 0$ and $m_1^A(0) = 0$. In this case $y(1, m^B) = 1$ for all m^B such that $m_v^B(1,1) = m^B$ for some $v \in X$. In order for the second expert's report to be informative, we need $y(0,1) > y(0,0) > y(0,-1)$. Thus, $m_1^B(0,0) = 1$ since $b > \frac{1}{2}$. We may also conclude $m_{-1}^B(-1,0) = m_0^B(-1,0) = -1$.

Now, we only need to determine $m_1^B(-1,0)$. First, $m_1^B(-1,0) \neq 1$ since $y(0,1) > y(0,0) = 0 > -1+b$. If $m_1^B(-1,0) = -1$, then $y(0,-1) = -\frac{1}{2}$. But $m_1^B(-1,0) = -1$ would not be optimal since $b \geq \frac{17}{21} > \frac{3}{4}$ implies $-1+b$ is closer to $y(0,0)$ than $y(0,-1)$.

If $m_1^B(-1,0) = 0$, then $y(0,-1) = -\frac{2}{5}$. Thus, $m_1^B(-1,0) = 0$ is optimal if $b \geq \frac{4}{5}$. Now, we compare the expected utility of a type 1 expert from $m_1^A(0) = 0$ and $m_1^A(0)$=1. By reporting the former, the expert induces actions $y(0,-1) = -y(0,1)$, $y(0,0)$, and $y(0,1)$ with equal probabilities. By reporting the latter, she induces actions $y(1, m_v^B(0,1))$ with equal probabilities for $v = -1, 0, 1$. In order for $m_1^A(0) = 0$ to be optimal, we need the difference in expected utility to be nonnegative. That is,

$$-\frac{1}{3}[(b - y(0,1))^2 + (b-0)^2 + (b - (-y(0,1)))^2] - (-\frac{1}{3}) \sum_v (b - y(1, m_v^B(0,1)))^2 \geq 0.$$

We need appropriate off-equilibrium behavior by the second expert and the decision maker in order for the above condition to be satisfied. Note that

$$\left|b - y(1, m_1^B(0,1))\right| \leq |b - y(1,1)| = 1 - b,$$

and that

$$\left|b - y(1, m_v^B(0,1))\right| \leq \max_{m^B} \left|b - y(1, m^B)\right| \leq \max\{b, b - y(1,-1)\}.$$

Thus it is necessary that either

$$-\frac{1}{3}[3b^2 + 2y(0,1)^2] - (-\frac{1}{3})[(1-b)^2 + 2b^2] \geq 0 \text{ and } y(1,-1) \geq 0$$

or

$$-\frac{1}{3}[3b^2 + 2y(0,1)^2] - (-\frac{1}{3})[(1-b)^2 + 2(b - y(1,-1))^2] \geq 0 \text{ and } y(1,-1) \leq 0.$$

The first inequality is impossible since $b > 1 - b > 0$. The second inequality on the other hand would imply $y(1,-1) \in [-\frac{2}{5}, 0]$ since $y(0,-1) \leq 0$. But an expert B of type -1 would prefer $y(1,-1)$ to $y(1,1) = 1$ in state 1 since it is closer to her most preferred action $1 - b$, due to our assumption $b \in [\frac{17}{21}, \frac{6}{7}]$. It is now a contradiction since I have shown above that $y(1, m^B) = 1$ for all m^B such that $m_v^B(1,1) = m^B$ for some $v \in X$. The above argument works as long as $b > \frac{3}{4}$.

Case 4. $m_1^A(-1) = 0$ and $m_1^A(0) = 1$. In this case $y(1, m^B) \geq 0$ for all m^B such that $m_v^B(s,1) = m^B$ for some $v \in X$ and $s \in S$.

In order to ensure that the second expert's reports be informative we must have $y(0,-1) < y(0,0)$ (which implies $y(0,0) < y(0,1)$ by symmetry) or $y(1,-1) < y(1,1)$.

1. $y(0,-1) < y(0,0)$.
   This immediately implies $m_1^B(0,0) = 1$, $m_1^B(1,0) = 1$, and $m_0^B(1,0) = 1$. We also know $m_1^B(-1,0) \neq 1$ since $y(0,0)$ is better than $y(0,1)$ in state -1 for an expert of type 1. From these facts we conclude $y(0,1) \geq \frac{2}{3}$. Hence $m_1^B(-1,0) = 0$ since a type 1 expert's most preferred action in state -1 is $-1 + b > -\frac{1}{3}$, which is closer to $y(0,0) = 0$ than to $y(0,-1)$. We conclude

$$y(0,1) = \frac{2}{3}.$$

(a) If $y(1,-1) = y(1,0) = y(1,1)$, they must all be equal to $\frac{2}{3}$. Thus it does not matter what the $m_v^B(s,1)$ are as long as they are such that $y(1,m^B) = \frac{2}{3}$ for all m^B such that $(1,m^B)$ is sent with positive probability. It remains to check whether $m_1^A(-1) = 0$ and $m_1^A(0) = 1$ are optimal. The strategy $m_1^A(-1) = 0$ is optimal since $m_1^A(-1) = 0$ induces actions $y(0,-1) = -\frac{2}{3}$ with probability $\frac{2}{3}$ and $y(0,0) = 0$ with probability $\frac{1}{3}$, while $m_1^A(-1) = -1$ induces action $y(-1,m^B) = -\frac{2}{3}$ for sure, and 0 is closer than $-\frac{2}{3}$ to the expert's favorite action $-1+b$ in state -1. The strategy $m_1^A(-1) = 1$ is even worse since it induces action $\frac{2}{3}$ for sure, which is worse than all the actions above as $b < 1$. The strategy $m_1^A(0) = 1$ is optimal since $m_1^A(0) = 1$ induces action $y(1,m^B) = \frac{2}{3}$ for sure, while $m_1^A(0) = 0$ induces actions $y(0,-1) = -\frac{2}{3}$, $y(0,0) = 0$, and $y(0,1) = \frac{2}{3}$ with equal probabilities, and 0 and $-\frac{2}{3}$ are farther than $\frac{2}{3}$ from her most preferred action b in state 0. This strategy profile thus constitutes an equilibrium. Note that this is exactly **strategy profile (B)**. In any strategy profile, the decision maker's expected payoff can be written

$$\sum_s \sum_{m^A} \sum_{m^B} P(s,m^A,m^B) u(y(m^A,m^B),s,0),$$

where $P(s,m^A,m^B)$ is the probability of the state-messages triple (s,m^A,m^B) occurring in equilibrium. Thus the decision maker's payoff in strategy profile (B) is

$$\begin{aligned} & -2 \cdot \tfrac{1}{3}\{\tfrac{2}{3}(\tfrac{2}{3}-1)^2 + \tfrac{1}{3}[\tfrac{2}{3}(\tfrac{2}{3}-1)^2 + \tfrac{1}{3}(0-1)^2]\} \\ & -\tfrac{1}{3}\{2 \cdot \tfrac{1}{3}(\tfrac{2}{3}-0)^2 + \tfrac{1}{3}[2 \cdot \tfrac{1}{3}(\tfrac{2}{3}-0)^2 + \tfrac{1}{3} \cdot 0]\} \\ = & -\tfrac{26}{81} \end{aligned}$$

(b) Now, we consider the possibility $y(1,-1) < y(1,1)$. Note that $y(1,1) = y(1,0)$ if the message pair $(1,1)$ is not sent in equilibrium, since otherwise we would have $m_1^B(1,1) = 1$. Similarly, $y(1,-1) = y(1,0)$ if the message pair $(1,-1)$ is not sent in equilibrium, since otherwise $m_{-1}^B(0,1) = -1$ is optimal as $b \geq \frac{17}{21} > \frac{1}{3}$, $y(1,0) \geq y(0,0) = 0$, and $y(1,-1) \geq y(0,-1) = -\frac{2}{3}$. Now we separate our discussion into two cases according to the number of different actions $y(1,m^B)$.

- $y(1,1) = y(1,0)$ or $y(1,-1) = y(1,0)$ (but not both).
  If $y(1,1) = y(1,0)$, it is possible that $(1,1)$ or $(1,0)$ (but not both) is never sent, but it does not matter to our discussion since we may

replace them with each other without essentially changing the strategy profile. Now we have $y(1,1) > \frac{2}{3} > y(1,-1)$, which implies $m^B_{-1}(1,1) = -1$ and $m^B_{-1}(0,1) = m^B_0(0,1) = -1$ as $b \geq \frac{2}{3}$. Hence $y(1,-1) \leq \frac{1}{2}$, which implies $m^B_1(0,1) \neq -1$ since $b \geq \frac{17}{21} > \frac{3}{4}$, $y(1,1) \leq 1$, and $y(1,-1) \leq \frac{1}{2}$. Thus, $y(1,1) = y(1,0) = \frac{4}{5}$ and $y(1,-1) = \frac{1}{2}$. The case $y(1,0) = y(1,-1)$ is similar, which results in $y(1,1) = \frac{4}{5}$ and $y(1,0) = y(1,-1) = \frac{1}{2}$.

Now, I check the optimality of $m^A_1(-1) = 0$ and $m^A_1(0) = 1$. First, note that $m^A_1(-1) = 0$ induces actions $y(0,-1) = -\frac{2}{3}$ with probability $\frac{2}{3}$ and $y(0,0)$ with probability $\frac{1}{3}$, that $m^A_1(-1) = -1$ induces actions $y(-1,-1) = -\frac{4}{5}$ with probability $\frac{2}{3}$ and $y(-1,1) = -\frac{1}{2}$ with probability $\frac{1}{3}$, and that $m^A_1(-1) = 1$ induces action $y(1,-1) = \frac{1}{2}$ for sure. Thus $m^A_1(-1) = 0$ is better than $m^A_1(-1) = -1$ since a type 1 expert prefers $-\frac{2}{3}$ to $-\frac{4}{5}$ and 0 to $-\frac{1}{2}$ in state -1, due to our assumption $b \in [\frac{17}{21}, \frac{6}{7}]$. The difference in expected utility between $m^A_1(-1) = 0$ and $m^A_1(-1) = 1$ is

$$-\frac{1}{3}[2(-\frac{2}{3}-(-1+b))^2+(0-(-1+b))^2]-(\frac{1}{2}-(-1+b))^2 = \frac{199}{36}-\frac{17}{3}b,$$

which is positive as long as $b < \frac{199}{204}$. Thus $m^A_1(-1) = 0$ is better than $m^A_1(-1) = 1$ given our assmptions. Second, $m^A_1(0) = 1$ induces actions $y(1,1) = \frac{4}{5}$ with probability $\frac{1}{3}$ and $y(1,-1) = \frac{1}{2}$ with probability $\frac{2}{3}$, while $m^A_1(0) = 0$ induces actions $y(0,1) = \frac{2}{3}$, $y(0,0) = 0$, and $y(0,-1) = -\frac{2}{3}$ with equal probabilities. Thus $m^A_1(0) = 1$ is a better response than $m^A_1(0) = 0$ since a type 1 expert prefers $\frac{4}{5}$ to $\frac{2}{3}$ and $\frac{1}{2}$ to any nonpositive action, due to our assumption $b \in [\frac{17}{21}, \frac{6}{7}]$. Thus, the strategy profile constitutes an equilibrium. Note that it is **strategy profile (C)**. In this strategy profile, the decision maker's expected payoff is

$$\begin{aligned}
&-2 \cdot \tfrac{1}{3}\{\tfrac{2}{3}[\tfrac{2}{3}(\tfrac{4}{5}-1)^2 + \tfrac{1}{3}(\tfrac{1}{2}-1)^2] + \tfrac{1}{3}[\tfrac{2}{3}(\tfrac{2}{3}-1)^2 + \tfrac{1}{3}(0-1)^2]\} \\
&-\tfrac{1}{3}\{2 \cdot \tfrac{1}{3}[\tfrac{2}{3}(\tfrac{1}{2}-0)^2 + \tfrac{1}{3}(\tfrac{4}{5}-0)^2] + \tfrac{1}{3}[2 \cdot \tfrac{1}{3}(\tfrac{2}{3}-0)^2 + \tfrac{1}{3} \cdot 0]\} \\
= \; &-\tfrac{104}{405}.
\end{aligned}$$

- $y(1,1) > y(1,0) > y(1,-1)$.

  This implies $(1,-1)$ must be sent in equilibrium as $m^A_1(0) = 1$ and $m^B_{-1}(0,1) = -1$ is uniquely optimal given $b \in [\frac{17}{21}, \frac{6}{7}]$ and $y(1,-1) \in [y(0,-1), y(1,0)) = [-\frac{2}{3}, y(1,0))$. Since $m^A_1(-1) \neq 1$, $y(1,m_B) \geq 0$ for

Table 2: Strategy Profile (D)

m^A	Type 0	Type 1
State -1	-1	0
State 0	0	1
State 1	1	1

m_0^B	$m^A=-1$	$m^A=0$	$m^A=1$
State -1	-1	-1	-1
State 0	1	0	-1
State 1	1	1	1

m_1^B	$m^A=-1$	$m^A=0$	$m^A=1$
State -1	1	0	-1
State 0	1	0	1
State 1	1	0	1

y	$m_B=0$	$m_B=1$
$m_A=-1$	$-2/3$	$-2/3$
$m_A=0$	0	$2/3$
$m_A=1$	$2/3$	$2/3$

all $(1, m_B)$ that are sent in equilibrium. Thus, $m_1^B(1,1) = m_0^B(1,1) = 1$ and $m_{-1}^B(0,1) = m_0^B(0,1) = -1$.
Therefore $y(1,-1) \leq \frac{1}{2}$, which implies that $m_1^B(0,1) \neq -1$ as $b \geq \frac{17}{21} > \frac{3}{4}$.
If $m_1^B(0,1) = 0$ then we need $|y(1,0) - b| \leq |y(1,1) - b|$, which implies $y(1,0) \geq 2b - y(1,1) \geq 2b - 1 > \frac{1}{2}$. The last inequality sign is due to our assumption that $b \in [\frac{17}{21}, \frac{6}{7}]$. This requires $m_{-1}^B(1,1) = 0$, since $m_1^B(1,1) = m_0^B(1,1) = 1$. But given $y(1,0) > \frac{1}{2}$, $m_{-1}^B(1,1) = 0$ is not optimal since $y(1,-1) \in [0, y(1,0))$ and $b \geq \frac{17}{21} > \frac{3}{4}$.
If $m_1^B(0,1) = 1$, then $y(1,0) = 1$ as long as $(1,0)$ is sent in equilibrium, which would violate $y(1,0) < y(1,1)$.
Thus, $(1,0)$ is never sent in equilibrium, and this case collapses into strategy profile (C).

2. $y(0,0) = y(0,1) = 0$ and hence $y(1,-1) < y(1,1)$ by informativeness. The analysis is similar to that above, and the only possible equilibrium involves $y(1,1) = \frac{4}{5}$ and $y(1,-1) = \frac{1}{2}$.
It remains to check the optimality of $m_1^A(0) = 1$ and $m_1^A(-1) = 0$. The strategy $m_1^A(0) = 1$ induces actions $y(1,1) = \frac{4}{5}$ with probability $\frac{1}{3}$ and $y(1,-1) = \frac{1}{2}$ with probability $\frac{2}{3}$, while $m_1^A(0) = 0$ induces action 0 for sure. All positive actions are preferred to 0 since $b > \frac{1}{2}$. Hence $m_1^A(0) = 1$ is optimal. The strategy $m_1^A(-1) = 0$ induces the action 0 for sure, $m_1^A(-1) = -1$ induces actions $y(-1,-1) = -\frac{4}{5}$ with probability $\frac{2}{3}$ and $y(-1,1) = -\frac{1}{2}$ with probability $\frac{1}{3}$, and $m_1^A(-1) = 1$ induces the action $y(1,-1) = \frac{1}{2}$ for sure. Among all these actions 0 is a type 1 expert's most preferred action in state -1 as $\frac{3}{4} < b < 1$.

Thus, $m_1^A(-1) = 0$ is optimal.

Note that this is exactly **strategy profile (D)**. In this strategy profile, the decision maker's expected payoff is

$$\begin{aligned} & -2 \cdot \tfrac{1}{3}\{\tfrac{2}{3}[\tfrac{2}{3}(\tfrac{4}{5}-1)^2 + \tfrac{1}{3}(\tfrac{1}{2}-1)^2] + \tfrac{1}{3}(0-1)^2\} \\ & -\tfrac{1}{3}\{2 \cdot \tfrac{1}{3}[\tfrac{2}{3}(\tfrac{1}{2}-0)^2 + \tfrac{1}{3}(\tfrac{4}{5}-0)^2] + \tfrac{1}{3} \cdot 0\} \\ = & -\tfrac{16}{45}. \end{aligned}$$

Case 5. $m_1^A(0) = 1$ and $m_1^A(-1) = 1$.

Again for any m^B such that $(0, m^B)$ is sent in equilibrium, we have $y(0, m^B) = 0$. This implies that $y(0, 1) = 0$. Otherwise $m_1^B(0, 0) = 1$ would be true as $b > \frac{1}{2}$. Now monotonicity requires $y(1, -1) \geq 0$ since $y(0, -1) = 0$. In order for expert B's opinion to be informative, we need $y(1, -1) < y(1, 1)$. So $m_v^B(-1, 1) \neq 1$ for any $v \in X$ since $b < 1$. Now $y(1, -1)$ must be negative, since $m_1^B(1, 1)$ and $m_0^B(1, 1)$ both belong to $\arg\max_{m^B} y(1, m^B)$ hence cannot be -1.

Summarizing the above arguments gives us the proposition. □

Proof of Lemma 4. First, I argue that Equation (2) implies $r_0^*(1, -1) = r_0^*(-1, 1) = r_0^*(0, 1) = r_0^*(0, -1) = 1$ and $r_1^*(1, -1) = r_1^*(0, -1) = 1$. Recall that the reviewer wants the decision to be as close to her most preferred action as possible. In all these cases, $|y_t^* - (s + b_v)| = \max_{t' \in S} |y_{t'}^* - (s + b_v)|$, and there exists $\tilde{t} \in S$ such that $|y_t^* - (s + b_v)| > \left|y_{\tilde{t}}^* - (s + b_v)\right|$. For example, when $v = 0, s = 0, t = 1$, $|y_1^* - (0 + 0)| = |-y_1^* - (0 + 0)| > |0 - (0 + 0)|$.

Second, Equation (2) implies that $r_0^*(s, s) = 0$ and $r_1^*(1, 1) = r_1^*(0, 1) = 0$ as in these cases

$$|y_t^* - (s + b_v)| = \min_{t' \in S} |y_{t'}^* - (s + b_v)|.$$

Simplifying Equation (2) further using condition (SE-3), I have $r_v^*(s, t) = 1$ if and only if

$$2\gamma(y_1^*)^2 + (s + b_v)^2 < (y_t^* - (s + b_v))^2. \quad \text{(S-1)}$$

Since $y_0^* = 0$ and $\gamma > 0$, L.H.S. $>$ R.H.S. for all s, v. Thus, $r_v(s, 0) = 0$ for all $v \in X, s \in S$. Since 0 is never rejected, $m_0(0) = 0$.

Now, I argue $m_0(1) = m_1(1) = 1$. First, by Lemma 3 and the proven results $r_0(1, -1) = r_1(1, -1) = 1$, they are not equal to -1. By reporting 0 in state 1, an expert of type $x \in \{0, 1\}$ receives an expected payoff of

$$-(0 - (1 + b_x))^2.$$

When reporting 1 in state 1, a reviewer of type 0 or 1 would accept it, but a reviewer of type -1 may reject it. Thus, the expert's expected payoff is greater than or equal to

$$\begin{aligned} & -P(v=0,1)(y_1^* - (1+b_x))^2 - P(v=-1)\textstyle\sum_{t\in S}(y_t^* - (1+b_x))^2 \\ = \; & -\tfrac{2}{3}(y_1^* - (1+b_x))^2 - \tfrac{1}{3}(2\gamma(y_1^*)^2 + (1+b_x)^2). \end{aligned}$$

The difference in utility between reporting 1 and reporting 0 is at least

$$2\cdot\frac{2}{3}(1+b_x)y_1^* - \frac{2}{3}(1+\gamma)(y_1^*)^2.$$

This expression is positive since $b_x \geq 0$, $y_1^* \in (0,1]$, and $\gamma < \frac{1}{2}$. Hence $m_1(1) = m_0(1) = 1$, which concludes the proof of part 2.

The only review decisions left to check are $r_1(-1,1)$ and $r_1(-1,-1)$. Substituting $v=1,\ s=-1,\ t=1$ into Equation (S-1), we get the condition for $r_1(-1,1)=1$ to be

$$0 \;<\; (1-2\gamma)(y_1^*)^2 + 2(1-b)y_1^*,$$

which holds for any $y_1^* > 0$. Hence $r_1(-1,1)=1$.

Similarly, by (S-1), $r_1(-1,-1)=1$ if and only if

$$0 \;<\; (1-2\gamma)(y_1^*)^2 - 2(1-b)y_1^*,$$

which holds only if

$$y_1^* > \frac{2(1-b)}{1-2\gamma}.$$

I claim $r_1(-1,-1)=0$ and prove it by contradiction. Suppose $r_1(-1,-1)=1$ instead. By Lemma 3, $m_1(-1)\neq -1$. As just shown, $r_1(-1,1)=1$, which together with Lemma 3 implies $m_1(-1)\neq 1$. Hence $m_1(-1)=0$.

Now we only need discuss $m_1(0)$. By Lemma 3 and the result that $r_1(0,-1)=1$, it can only be either 0 or 1.

First, consider the possibility $m_1(0)=0$. Reporting 0 guarantees an expected utility of $-(0-b)^2$, while reporting 1 induces rejection from reviewers of types 0 and -1 and acceptance from reviewers of type 1, leading to an expected utility of $-\frac{2}{3}[2\gamma(y_1^*)^2+b^2]-\frac{1}{3}(y_1^*-b)^2$. The difference in utility between reporting 1 and reporting 0 is

$$2b(1-\frac{2}{3})y_1^* - [1-\frac{2}{3}(1-2\gamma)](y_1^*)^2.$$

We need the above expression to be less than or equal to 0, which translates into

$$y_1^* \geq \frac{2b(1-\frac{2}{3})}{1-\frac{2}{3}(1-2\gamma)}.$$

In this strategy profile,

$$\begin{aligned}\gamma &= P(s=1)P(x=0 \text{ or } 1)[P(v=0 \text{ or } 1)+P(v=-1)\gamma] \\ &\quad +P(s=-1)P(x=0 \text{ or } -1)P(v=1)\gamma \\ &= \tfrac{1}{3}\times\tfrac{2}{3}[\tfrac{2}{3}+\tfrac{1}{3}(2\gamma)]\end{aligned}$$

From the above expression, we derive $\gamma = 4/23$. Furthermore $y_1^*\gamma = P(s=1, m=1) - P(s=-1, m=1) = \frac{1}{9}(1+\frac{1}{3})$. Hence, $y_1^* = 23/27$. But, these values violate the preceding inequality because $b \in [\frac{17}{21}, \frac{6}{7}]$ (hence $b > \frac{13}{18}$), making $m_1(0) = 0$ not optimal.

If $m_1(0) = 1$, then

$$\begin{aligned}\gamma &= P(s=0)P(x=1)[P(v=1)+P(v=0 \text{ or } -1)\gamma] \\ &\quad +P(s=0)P(x=-1)P(v=0 \text{ or } 1)\gamma \\ &\quad +P(s=1)P(x=0 \text{ or } 1)[P(v=0 \text{ or } 1)+P(v=-1)\gamma] \\ &\quad +P(s=-1)P(x=0 \text{ or } -1)P(v=1)\gamma \\ &= \tfrac{1}{3}\times\tfrac{1}{3}[1-\tfrac{2}{3}(1-\gamma)]+\tfrac{1}{3}\times\tfrac{2}{3}[\tfrac{2}{3}+\tfrac{1}{3}(2\gamma)] \\ &= \tfrac{1}{9}+\tfrac{2}{9}[1-\tfrac{2}{3}(1-2\gamma)]\end{aligned}$$

From this we obtain $\gamma = \frac{\frac{1}{9}(1+2\frac{1}{3})}{1-\frac{4}{9}(1-\frac{1}{3})}$. Again $\gamma y_1^* = \frac{1}{9}(1+\frac{1}{3})$. Substituting this into the condition for $r_1(-1,-1) = 1$, I have $r_1(-1,-1) = 1$ if and only if

$$\frac{1}{3} < \frac{\frac{1}{3} - 2(1-b)}{4(1-b) - \frac{1}{3}}.$$

But, when $b \in [\frac{17}{21}, \frac{6}{7}]$ (hence less than $\frac{13}{15}$), the expression on the right hand side is less than or equal to $\frac{1}{3}$, contradiction. Hence $r_1(-1,-1) = 0$ in equilibrium.

Summarizing the above arguments gives us Lemma 4. □

Proof of Lemma 5. The expected utility from acceptance is $-(y_m - (s+b_v))^2$, and that from rejection is $-\sum_{m'} \gamma_{m'}(y_{m'} - (s+b_v))^2$, where $\gamma_{m'}$ indicates the probability of message m' being received in equilibrium. The difference in expected utility between rejection and acceptance is

$$-[\sum_{m'} \gamma_{m'}(y_{m'} - (s+b_v))^2] - (y_m - (s+b_v))^2 = -[\sum_{m'} \gamma_{m'} y_{m'}^2 - y_m^2 + 2y_t(s+b_v)].$$

In deriving the above equality, I have used the fact that $\sum_{m'} \gamma_{m'} y_{m'} = 0$. Thus holding everything else fixed, an increase in v increases b_v, which decreases the difference in utility if $y_m > 0$. This makes it less likely for the expert to reject the message. The contrary is true if $y_m < 0$. Hence the first two statements.

If $y_m = 0$ then the difference in utility is always negative unless $y_{m'} = 0$ for all $m' \in M$, which is not informative and which implies rejection is meaningless. Thus the expert should always accept a message m inducing the action 0. $\square$