

# Combining Expert Opinions <sup>\*</sup>

Ming Li<sup>†</sup>

January, 2009

## Abstract

I analyze a model of advice with two perfectly informed experts and one decision maker. The bias of an expert is her private information. I show that consulting two experts is better than consulting just one. In the simple “hierarchical review” mechanism, the decision maker receives just one report, and the second expert decides whether to block the first expert’s report. Simultaneous consultation transmits information better than sequential consultation and hierarchical review. However, hierarchical review still achieves significant information transmission, with the decision maker receiving only one report. There is an asymmetric equilibrium that is more efficient than the symmetric equilibrium. When given the chance to discover biases of experts, the decision maker may prefer not to do so.

**JEL codes:** C72, D72, L22.

**Keywords:** Expert opinions, strategic information transmission, multiple experts.

---

<sup>\*</sup>This paper grew out of a chapter of my doctoral thesis. I am indebted to my advisor, Larry Samuelson, for advice and encouragement. I am also grateful to Bill Sandholm for very useful suggestions. Helpful comments from Michael Baye, Thomas Gilligan, Eric Rasmusen, and seminar participants at Indiana University (BEPP), University of Wisconsin-Madison, Stony Brook Game Theory Conference, WEA Meeting in Taipei, and PET Conference in Hanoi have been instrumental in the improvement of this paper. I am also grateful to the editor and two anonymous referees for very constructive comments and suggestions. Financial support from a Concordia University Startup Grant is gratefully acknowledged. Any remaining mistakes are my own.

<sup>†</sup>Department of Economics, Concordia University, Montreal. Email: mingli@alcor.concordia.ca.

# 1 Introduction

People specialized in decision making rarely have specialized knowledge about things on which they make decisions. They usually turn to experts for advice. This naturally brings about the issue of objectivity and credibility of advice from the experts. As well said by Samuel Butler, “opinions have vested interests just as men have.” Experts have more specialized knowledge, but their preferences on issues may well be different from those of the decision maker. If the US President asks his cabinet members on policies about world affairs, they are likely to recommend policies that favor their own material interests or ideological agenda.<sup>1</sup> If a CEO elicits advice on compensation schemes, each employee is likely to favor schemes that benefits her most.

A common way believed potentially to alleviate the problem is to seek second opinions. By asking multiple experts, it is conceivable that decision makers may be able to hear more diversified opinions, so as to get a better picture of the underlying situation and avoid blunders in the final decision.

However, the decision maker faces new difficulties. An expert takes the presence of other experts into strategic consideration when providing advice. The decision maker must first listen to these experts, disentangle the strategic elements in the experts’ advice, and choose the best action. As an outsider to the experts’ specialized field, the decision maker may not know the actual preferences of the experts. A second problem is that interactions among experts are complicated. Experts may strengthen, rebut, and fine tune each other’s reports depending on their respective biases. The decision maker may not have the time or energy to listen to and handle all the pros and cons on an issue.

Thus, it is important to find the kind of communication mechanisms the decision maker should use to maximize information transmission efficiency given time and financial constraints.

In the business organization environment, lower level managers and employees have better information about consumer preferences, production technologies, demand for staff, etc. A CEO has to choose a mechanism to combine opinions from employees. He does not necessarily know their preferences. The reason is two-fold: on the one hand, different individuals may have different biases, which are hard for top management to track; on the other hand, the issues to be dealt with are varied, people’s biases may change depending on the issue at hand. If one views communication mechanisms as part of the organization structure, which is relatively inflexible,

---

<sup>1</sup>In this paper, I refer the decision maker as “he,” and each expert “she.”

then it is important to have a mechanism that optimizes communication allowing uncertainty about experts' biases.

In this paper, I study efficiency of information transmission between experts and the decision maker in various communication mechanisms. Efficiency can be measured by the decision maker's expected utility. For the class of preferences I consider, before knowing the state, any expert ranks equilibria the same as the decision maker does.

In my model, there are two perfectly informed experts and a single decision maker. The experts may have biases on what decision to take, and their biases are unknown to both the decision maker and the other expert.

The decision maker has the option to ask just one expert or ignore the report from one of the experts. On the other hand, when using information from both experts, he has various options.

One mechanism he can choose is to ask each expert *simultaneously*. Experts are aware of the existence of another expert, but they do not observe each other's report when making their own reports. Another option for the decision maker is to ask two experts *sequentially*. The first expert makes a report, and the second expert gets to observe it and then offer her own report. Hearing both reports, the decision maker makes a decision. I call these two mechanisms direct consultation mechanisms since the decision maker hears both experts' reports. I also consider a mechanism called *hierarchical review*, in which one of the experts does not report her own opinion, but decides whether to pass on the other expert's report. If she accepts the expert's report, then the decision maker hears the original report. If she rejects it, however, the decision maker receives a random signal. The distribution of the signal is endogenously generated by interactions between the reviewer and the expert, in a sense to be made clear later.

Note that, in the hierarchical review mechanism, the decision maker only receives one final report, as opposed to two reports in the direct consultation mechanisms. Therefore, this mechanism is simpler for the decision maker and presumably less costly. In some sense, he delegates part of the information solicitation task to the reviewer.

The consideration of the hierarchical review mechanism is of interest since it captures some of the features of many communication and organization structures in reality. CEOs do not directly listen to reports of lower-level employees. Lower-level employees make reports to the mid-level management, and what the executives get is a selective pool of reports that are filtered by mid-level management. Similarly, the US president learns information through his advisors instead of listening directly

to other experts.

**Findings.** In the hierarchical review mechanism, in symmetric equilibria, truthful reports are never rejected by reviewers, so review deters the expert from lying and keeps false reports from reaching the decision maker when she lies.<sup>2</sup>

Consulting two experts is better than consulting just one. Consulting two experts simultaneously does the best among all three mechanisms. When comparing hierarchical review with consulting two experts sequentially, there exist symmetric equilibria in which sequential consultation does better than hierarchical review. However, there also exist equilibria in which the decision maker is worse off than under hierarchical review.

Although the setup of the model is symmetric, there also exist asymmetric equilibria in the hierarchical review case. Furthermore, the decision maker is better off in some asymmetric equilibria than under symmetric ones. Finally, I show that given the chance to discover what the biases of experts are, the decision maker may prefer not to do so.

The paper proceeds as follows. Section 2 introduces the basics of the model. Section 3 characterizes symmetric equilibria of the various communication mechanisms. Section 4 compares welfare under the three different mechanisms. Section 5 discusses extensions of the basic model. Section 6 summarizes related literature. Section 7 concludes and suggests further research. Proofs can be found in the appendix unless otherwise noted.

## 2 The Model

Two experts advise a decision maker. The decision maker takes an action that affects payoffs of both himself and the experts. The decision maker's objective is to maximize the welfare of a society or an organization. His optimal action depends on some underlying state. The decision maker does not know the state, but the experts do. The experts' preferences over actions may be different from those of the decision maker. Their preferences are unknown to the decision maker and to each other.

There are three possible states of the world. The state space is  $S = \{-1, 0, 1\}$ . Each state happens with probability  $1/3$ . The decision maker's optimal action in state  $s \in S$  is equal to  $s$ . The decision maker can take any action from the set  $Y = [-1, 1]$ .

---

<sup>2</sup>Since the decision taken by the decision maker does not correspond exactly to the label of messages, I use the words "lie" and "misrepresentation" to mean any report by biased-experts that is different from what an unbiased expert would send.

There are three types of experts from the set  $X = \{-1, 0, 1\}$ , each with probability  $1/3$ . The state and both experts' biases are independent of one another. An expert of type  $x \in X$  has bias

$$b_x = bx,$$

where  $b$  satisfies the assumption below.<sup>3</sup>

**Assumption 1.** The bias value  $b$  lies in  $[17/21, 6/7]$ .

Thus, there exist unbiased experts and experts with biases to the two extremes. Each expert's bias is her private information.

Both the experts and the decision maker have quadratic-loss utility functions:

$$u(y, s, \tilde{b}) = -(y - (s + \tilde{b}))^2, \tag{1}$$

where  $y$  is the action taken by the decision maker,  $s$  is the true state, and  $\tilde{b}$  is the bias of the agent. For the decision maker  $\tilde{b} = 0$ , and for an expert of type  $x \in X$ ,  $\tilde{b} = b_x$ . This function is the same as that in the special case introduced by Crawford and Sobel (1982). It has the convenient property that for any agent with bias  $\tilde{b}$ , the ideal action to take is  $E(s) + \tilde{b}$ , the expected value of the state plus the bias. An expert with a right bias always prefers the decision maker to take an action greater than the true state, and vice versa.

Each expert is allowed to send a signal from a message space  $M$ . For tractability, I limit  $M$  to be  $S = \{-1, 0, 1\}$ .<sup>4</sup>

---

<sup>3</sup>The results change when I vary  $b$ . First, there exists a fully revealing equilibrium for all communication mechanisms above when  $b \leq 1/2$ , in which every expert tells the truth and the decision maker takes the corresponding actions. The comparisons between different mechanisms may change when  $b$  is outside the range of values in Assumption 1. For example, when  $b = 2/3$ , sequential consultation gives the most informative equilibrium, followed in turn by hierarchical review and simultaneous consultation. Finally, the limit to three messages affects the equilibrium outcomes when  $b$  is relatively small, as without this limit there would exist equilibria where more than three messages are sent.

<sup>4</sup>This is *not* without loss of generality. There are scenarios in which allowing additional messages may improve communication between the experts and the decision maker. However, even in richer environments than this model, experts are sometimes limited to relatively small message spaces due to either conventions or the decision maker's information processing constraints. For example, stock analysts overwhelmingly adopt the categorized ranking system, presumably to make their recommendations easier to comprehend for investors. Since the restriction applies to all mechanisms, I put all the mechanisms on a level "playing field." Notably, Austen-Smith (1993) and Morris (2001) also consider models with discrete spaces and restrict the message space to be equal to the state space.

Except from taking an action according to the report(s) he receives, the decision maker cannot commit to decision rules using any other device. In particular, he cannot use monetary transfers that are correlated with reports.

The decision maker asks for advice from experts. He can choose to ask just one expert. In the case where the decision maker asks both experts, he may choose among various mechanisms. In this paper, I consider the following three: simultaneous consultation, sequential consultation, and hierarchical review.

## Simultaneous Consultation

In the *simultaneous consultation* mechanism, the decision maker makes a decision after hearing simultaneous reports from the two experts.

Each expert is allowed to send a signal from the message space  $M$ . For clarity of notation, I label the experts  $A$  and  $B$ . Expert  $i$ 's report is denoted  $m^i$ ,  $i = A, B$ . Define the strategy of expert  $i$  of type  $x$  as  $m_x^i : S \rightarrow M$ , for  $i = A, B$  and  $x \in X$ . In other words,  $m_x^i(s)$  is what expert  $i$  of type  $x$  would report if the state is  $s \in S$ . The decision maker's strategy is defined as  $y : M \times M \rightarrow [-1, 1]$ , where  $y(m^A, m^B)$  is the action taken by the decision maker when he receives the message pair  $(m^A, m^B)$ .

## Sequential Consultation

In the *sequential consultation* mechanism, expert A makes a report; expert B observes it and makes her own report; finally, the decision maker hears *both* reports and makes a decision.

With a slight abuse of notation (since  $m_x^B$  has already been used above as a function with a single argument), their strategies can be defined respectively as  $m_x^A : S \rightarrow M$ , and  $m_x^B : S \times M \rightarrow M$ . For A,  $m_x^A(s)$  is the report sent by an expert of type  $x$  when the true state is  $s$ ; for B,  $m_x^B(s, t)$  is the report sent by an expert of type  $x$  when the true state is  $s$  and expert A has reported  $t$ . The decision maker's strategy is  $y : M \times M \rightarrow [-1, 1]$ , where  $y(m^A, m^B)$  represents the action taken when the message pair is  $(m^A, m^B)$ .

## Hierarchical Review

In the *hierarchical review* mechanism, expert A makes a report about the underlying state, but her report must pass through a reviewer (expert B) before reaching the decision maker. The reviewer may reject the report or accept it. When she rejects it, the decision maker receives a random report coming from the endogenous distribution

of signals generated by interactions between the experts. The decision maker cannot distinguish whether the message he receives is an original report or a random report drawn after a rejection. He takes an action based solely on the report he eventually receives.

In this mechanism, the random distribution of reports can be interpreted as being generated by a population of decision makers and experts facing identical uncertain situations.<sup>5</sup> Further, this mechanism is somewhat similar in spirit to mechanisms involving “veto power” in the cheap talk literature.<sup>6</sup> In these mechanisms, the decision maker retains the power to “do nothing,” that is, maintains the status quo. The results in these models typically depend on the exogenously given value of the status quo. To evaluate the overall efficiency of a mechanism with veto power, a distribution over the status quo should be specified. In the hierarchical review mechanism studied in this paper, the distribution is endogenously generated by the mechanism itself.

Denote by  $\Gamma$  the distribution of messages eventually received by the decision maker in equilibrium. Let  $\gamma_m$  denote the probability of message  $m \in M$  being received. If the expert’s advice gets *rejected*, then the decision maker receives a message randomly drawn from the endogenous distribution  $\Gamma$ .

Let the expert’s strategy be  $m_x : S \rightarrow M$ , where  $m_x(s)$  is the message sent by an expert of type  $x \in X$  when the state is  $s$ . Let  $\tilde{m}_x(s, t) = 1_{m_x(s) = t}$ . Let the reviewer’s strategy be  $r_v : S \times M \rightarrow \{0, 1\}$ , where  $r_v(s, t)$  indicates whether a reviewer of type  $v$  rejects message  $t$  when the state is  $s$ . The value 1 means rejection, and 0 means acceptance. The decision maker bases his decision on the message he receives as a result of the expert’s advice and the reviewer’s decision. The decision maker’s strategy is thus defined as  $y : M \rightarrow [-1, 1]$ . For convenience denote by  $y_m$  the action taken by the decision maker after hearing message  $m \in M$ .

---

<sup>5</sup>Alternatively, one may let the distribution of the random report be exogenously given. Blume, Board, and Kawamura (2007) adopt the uniform noise distribution in their model, where the expert’s report could exogenously fail to reach the decision maker, in which case the decision maker receives a message drawn from the noise distribution.

<sup>6</sup>See Gilligan and Krehbiel (1987), Krishna and Morgan (2001a), Dessein (2002), Mylovanov (2008), and Board and Dragu (2006).

## Equilibrium

For tractability, I consider only pure strategy equilibria.<sup>7</sup> An *equilibrium* is a strategy profile that satisfies the following conditions where they apply:

(EQ1) An expert of any type  $x \in X$  sends the message that maximizes her expected utility in any state  $s \in S$ , given strategies of the other expert, the decision maker, and the distribution  $\Gamma$ .

(EQ2) Hierarchical review: A reviewer of any type  $v \in X$  rejects a message  $t$  in state  $s$ , i.e.  $r_v^*(s, t) = 1$  if and only if rejection gives her higher expected utility than acceptance, given the decision maker's strategy, and the distribution  $\Gamma$ . That is,

$$u(y_t, s, v) < \sum_{t' \in M} \gamma_{t'} u(y_{t'}, s, v).$$

(EQ3) The decision maker takes action  $y(m) = E(s|m)$  when receiving message(s)  $m$ , where  $m$  could either be a scalar or a two-dimensional vector.

(EQ4) Hierarchical review: the distribution of messages generated from interactions between experts and reviewers is the same as the distribution from which a random message is drawn when a report is rejected and successfully blocked. Formally, this can be written as

$$\begin{aligned} \gamma_t = & \sum_{s \in S} P_s \sum_{x \in X} P_x \tilde{m}_x(s, t) \sum_{v \in X} P_v [1 - r_v(s, t)] \\ & + \sum_{s \in S} P_s \sum_{x \in X} P_x \sum_{t' \in M} \tilde{m}_x(s, t') \sum_{v \in X} P_v r_v(s, t') \cdot \gamma_{t'}, \end{aligned}$$

where  $P_s$ ,  $P_x$ , and  $P_v$  stand for the probabilities of state  $s$ , type  $x$  expert, and type  $v$  reviewer occurring respectively. They are all equal to  $1/3$  in this model. The first part is the probability of the event that experts report the message  $t$  and it gets through to the decision maker, and the second part is the probability of the event that experts report any message, the message gets rejected, and the randomly drawn message is  $t$ .

As in all cheap talk models, two issues arise. First, the meaning of messages. I make the following assumption to reduce essentially identical equilibria into one.<sup>8</sup>

---

<sup>7</sup>This is without loss of generality for a single expert and expert  $B$  in the sequential mechanism, as  $b$  is less than one, which precludes mixing by any expert. There may however exist equilibria in mixed strategies for other mechanisms.

<sup>8</sup>This assumption does not pose additional restrictions for simultaneous consultation and hierarchical review. However, it does for sequential consultation, though I am not aware of any non-monotonic equilibrium informationally superior to monotonic ones.

The idea is that a high message should more likely indicate a higher state than a low message. A right-biased expert should be more likely to make a right-biased report than other types of experts. An expert should be more likely to report a state to be high when it is indeed high.

**Assumption 2.** (*Monotonicity.*) The decision maker's strategy must be increasing in the message(s) he receives. The experts' reports should be increasing in the state and their biases.

Second, multiplicity of equilibria. In particular, a babbling equilibrium always exists. Following previous work applying the cheap-talk model, I will focus on the most *informative* within all symmetric equilibria, which is defined as follows.<sup>9</sup>

An equilibrium is *symmetric* if and only if the equilibrium strategy profile is a mirror image of itself. A pure strategy profile  $(\hat{m}, (\hat{r}), \hat{y})$  is a *mirror image* of another strategy profile  $(m, (r), y)$  if for all  $i = A, B$ ,  $x, v \in X$ ,  $s, t \in S$ , and  $m \in S$  or  $S \times S$ , the following conditions are satisfied where they apply:

- (SE1) Simultaneous consultation:  $m_x^i(s) = -\hat{m}_{-x}^i(-s)$ ;  
 Sequential consultation:  $m_x^A(s) = -\hat{m}_{-x}^A(-s)$ ;  
 Peer review or consulting one expert:  $m_x(s) = -\hat{m}_{-x}(-s)$ .
- (SE2) Sequential consultation:  $m_x^B(s, t) = -\hat{m}_{-x}^B(-s, -t)$ ;  
 Hierarchical review:  $r_v(s, t) = \hat{r}_{-v}(-s, -t)$ .
- (SE3)  $\hat{y}(m) = -y(-m)$ .
- (SE4) Peer review:  $\gamma_t = \hat{\gamma}_{-t}$  for all  $t \in M$ .

Intuitively, in a symmetric equilibrium experts and reviewers of type 1 and  $-1$  behave in a similar way, and state values  $-1$  and  $1$  and reports  $-1$  and  $1$  are treated in a similar way. When I make descriptions or prove facts about symmetric equilibria, I need only consider behavior of experts (and reviewers) of types 0 and 1.

Before characterizing the equilibria, observe that full revelation is not possible in equilibrium. This will be clear in the equilibrium characterization section. Here, I offer only an informal argument for the hierarchical review mechanism in place of

---

<sup>9</sup>Chen, Kartik, and Sobel (2008) provide an equilibrium selection criterion that justifies the focus on the most informative equilibrium in Crawford and Sobel's model with complete information about the expert's bias, though it remains unclear how their result can be generalized to models with uncertainty about biases. Other papers on refinements of cheap talk equilibria include those by Farrell (1993), Farrell and Rabin (1996), and Matthews, Okuno-Fujiwara, and Postlewaite (1991).

a formal proof. Due to the quadratic loss utility function, no reviewer would reject the report 0, because it would generate a random message symmetrically distributed over  $-1$ ,  $0$ , and  $1$ . Thus, a right-biased expert would prefer to report  $-1$  as  $0$ , ruling out full revelation as an equilibrium outcome.

### 3 Symmetric Equilibrium

In this section, I characterize the most informative equilibrium for each mechanism.

#### Consulting One Expert

**Proposition 1.** *When the decision maker consults only one expert, the only symmetric equilibrium is as follows:*

- 1)  $m_0^*(s) = s$ ,  $m_1^*(s) = s + 1$  if  $s \neq 1$ , and  $m_1^*(1) = 1$ ;
- 2)  $y_m^* = (2/3)m$ .

*In equilibrium, the decision maker's expected payoff is  $-10/27$ .*

In equilibrium, biased experts always misrepresent the state when possible. That is, an expert of type 1 reports state  $-1$  as  $0$  and  $0$  as  $1$ . They are able to do so since there are no forces to counteract or punish biased reports. In a sense, this is the worst that could happen to the decision maker in an informative equilibrium. Now, I investigate whether the introduction of another expert improves the situation.

#### Simultaneous Consultation

In this mechanism, each expert simultaneously sends a report to the decision maker. In addition to the symmetry conditions above, I add another symmetry condition.

**Assumption 3.** (*Anonymity.*)  $m_x^A(s) = m_x^B(s)$  for all  $x \in X$  and  $s \in S$ .

The idea behind this condition is that an expert's reports are not affected by her labelling, but only by the underlying state and her bias. As a result, in equilibrium, the decision maker's action is based only on the combination of message pairs, but not on the source of messages. The main result is as follows.

**Proposition 2.** *In the simultaneous consultation game, strategy profile (A), as defined in Table 1, is the only pure strategy symmetric equilibrium satisfying anonymity. In this equilibrium, the decision maker's expected payoff is  $-94/405$ .*

Table 1: Strategy Profile (A)

$m^A, m^B$	Type 0	Type 1	$y$	$m_B = 0$	$m_B = 1$
State -1	-1	0	$m_A = -1$	-2/3	0
State 0	0	1	$m_A = 0$	0	2/3
State 1	1	1	$m_A = 1$	2/3	4/5

Table 2: Strategy Profile (C)

$m^A$	Type 0	Type 1	$m_0^B$	$m^A = -1$	$m^A = 0$	$m^A = 1$
State -1	-1	0	State -1	-1	-1	-1
State 0	0	1	State 0	1	0	-1
State 1	1	1	State 1	1	1	1
$m_1^B$	$m^A = -1$	$m^A = 0$	$m^A = 1$	$y$	$m_B = 0$	$m_B = 1$
State -1	1	0	-1	$m_A = -1$	-4/5	-1/2
State 0	1	1	1	$m_A = 0$	0	2/3
State 1	1	1	1	$m_A = 1$	4/5	4/5

Strategy profile (A) is a “replication” of the equilibrium of the one-expert case. Here, biased reports are sometimes balanced by the other expert of a different bias. For example, although a right-biased expert would still report state  $-1$  as  $0$ , it is offset by the other expert when the other expert is of bias  $-1$  or  $0$ , which happens with probability  $2/3$ . When the decision maker receives the message pair  $(0, -1)$  or  $(-1, 0)$ , he takes the action  $-2/3$ . On the other hand, in the one-expert mechanism, he takes action  $0$  when he receives the message  $0$ , which is farther from his most preferred action  $-1$  in state  $-1$ .

## Sequential Consultation

In the sequential consultation mechanism, the second expert makes a report based on what the first expert has reported. The main result is as follows.

**Proposition 3.** *Strategy profiles (C), as defined in Table 2, is the most informative monotonic symmetric equilibria of the sequential consultation game. The decision maker’s expected payoff is  $-104/405$ .*

Table 3: Strategy Profile (E)

		$m^A$			
		Type 0	Type 1		
State -1		-1	0		
State 0		0	1		
State 1		1	1		

  

$r_0$	$m = -1$	$m = 0$	$m = 1$	$r_1$	$m = -1$	$m = 0$	$m = 1$
State -1	0	0	1	State -1	0	0	1
State 0	1	0	1	State 0	1	0	0
State 1	1	0	0	State 1	1	0	0

Now I examine the equilibrium more closely. In equilibrium, expert A always distorts her report towards the direction of her bias. For example, an expert of type 1 reports  $-1$  as  $0$  and  $0$  as  $1$ . Expert B acts as if she were the first expert and distorts her report towards her bias, if expert A has not made a biased report. If expert A has, then if expert B has the same bias as expert A, she may further distort it, make a moderate report, or correct the distortion by expert A it proves to be excessive (for example, reporting  $-1$  as  $1$ ). If she is unbiased or if her bias is opposite to expert A's, then she chooses to offset the distortion by expert A if this option is available in equilibrium. For example, an expert B of type 0 would like to report  $1$  if expert A has reported  $0$  as  $-1$ .

Sequential consultation also allows other symmetric monotonic equilibria, which are ex ante Pareto inferior to (C).<sup>10</sup>

## Hierarchical Review

In the hierarchical review mechanism, the reviewer decides whether to reject the expert's report in favor a random one, or to pass it on. Formally, when the report of an expert is rejected by a reviewer, the reviewer will draw a signal from the endogenous symmetric distribution,  $(\gamma, 1 - 2\gamma, \gamma)$ , where  $\gamma \in (0, 1/2)$ .

**Proposition 4.** *For the hierarchical review mechanism, strategy profile (E), as defined in Table 3, is the only symmetric equilibrium of the game ( $y_1^* = 46/63$  and  $\gamma = 7/23$ ). The decision maker's expected payoff is  $-194/567$ .*

<sup>10</sup>Notably, there are also equilibria in which monotonicity is violated and in which the decision maker does even worse than in the single-expert case. See Li (2008) for a detailed illustration.

Table 4: Comparisons of Equilibria: Decision Maker’s Payoff

	Absolute Payoff	Improvement Percentage
Babbling	$-2/3$	0
One Expert	$-10/27$	68.2
Hierarchical Review	$-194/567$	74.7
Sequential	$-104/405$	94.3
Simultaneous	$-94/405$	100

In the above equilibrium, the expert follows what she does in the one-expert mechanism. But, if she is biased, her misrepresentation in state 0 is rejected by a reviewer of the opposite bias or no bias. Rejection happens only in cases when an unbiased reviewer would also like to reject the report. Since the unbiased reviewer has the same preferences as the decision maker, such rejections improve the payoff of the decision maker.

## 4 Comparisons

In this section, I discuss the welfare comparisons between the communication mechanisms considered above. Table 4 offers a summary both in terms of absolute payoffs and relative improvement from the babbling outcome. Since von Neumann-Morgenstern expected utility is unique up to affine transformations, the second column accurately reflects the welfare comparisons between different equilibria.

Clearly, all two-expert mechanisms do better than the one-expert mechanism, conforming to the intuition that second opinions improve information transmission.

Among the two-expert mechanisms, simultaneous consultation yields the highest payoff to the decision maker, followed by sequential consultation (in its most informative equilibrium), and hierarchical review. However, sequential consultation generates multiple equilibria, some of which provide lower expected payoff than does hierarchical review.

**Result of comparisons.** *Considering the most informative equilibrium, the ranking of information transmission efficiency of the three mechanisms is (from the highest to the lowest): 1. simultaneous consultation; 2. sequential consultation; 3. hierarchical review. However, there exist*

*equilibria in sequential consultation in which the decision maker is worse off than he is under hierarchical review.*

In this paper, I model hierarchical review as a mechanism where interactions between experts are not transparent to the decision maker and only the final report is observable to the decision maker. This corresponds to economic situations in which the decision maker just receives one unified recommendation, instead of hearing each expert's opinion and what they think about each other's opinion. For example, a political leader would only choose an economic policy proposal, without knowing how the proposal has been promoted to prominence. It is reasonable to expect that monitoring interactions among experts is costly. If such costs are higher than the improvement in welfare from one expert to two experts with direct consultation, it may be better to use hierarchical review instead.

## 5 Discussions

### Asymmetric Equilibrium

Although symmetric equilibria seem to be a natural choice for the game given the symmetric setup of the model, I show herein that there exist asymmetric equilibria that are more informative than the symmetric equilibrium.

Let us first define strategy profile (F):

$$(F1) \quad m_{-1}(-1) = m_{-1}(0) = -1, \quad m_{-1}(1) = 0 \quad \text{and} \quad m_0(s) = m_1(s) = s \quad \text{for all } s \in S;$$

$$(F2) \quad r_{-1}(s, t) = 1 \quad \text{for } (s, t) = (-1, 0), (-1, 1), (0, 1) \quad \text{and } (1, -1), \quad \text{and } 0 \quad \text{otherwise};$$

$$r_0(s, t) = 1 \quad \text{for } (s, t) = (-1, 1), (0, -1), (0, 1) \quad \text{and } (1, -1), \quad \text{and } 0 \quad \text{otherwise};$$

$$r_1(s, t) = 1 \quad \text{for } (s, t) = (-1, 1), (0, -1) \quad \text{and } (1, -1), \quad \text{and } 0 \quad \text{otherwise};$$

$$(F3) \quad y_{-1} = -23/30, \quad y_0 = 23/63, \quad \text{and} \quad y_1 = 23/27;$$

$$(F4) \quad \gamma_{-1} = 10/23, \quad \gamma_0 = 7/23, \quad \text{and} \quad \gamma_1 = 6/23.$$

**Proposition 5.** *Strategy profile (F) (and its mirror image) is an equilibrium, which yields higher payoff for the decision maker than (E) does.*

In the asymmetric equilibrium, only the left-biased type misrepresents the states. A right-biased expert does not because, when she tells the truth, the decision maker's action is so close to her most preferred action that she finds misrepresentation worse

than truth-telling, taking into account the possibility of rejection. In contrast, the left-biased expert has stronger incentives to lie since lying induces a much more favorable decision than telling the truth, despite the risk of rejection. When receiving 0, the decision maker actually takes a positive action. The gain from having this message instead of an impartial one is that it reduces the amount of (in this case eliminates) misrepresentation by right-biased experts and it also reduces the loss from left-biased experts reporting state 1 as 0. Compared with the symmetric equilibrium, the loss from having a biased 0 is more than offset by the reduction in the loss from experts misrepresenting the states.

This somewhat counterintuitive result echoes with that found by Admati and Pfleiderer (2004) under a different setup. Morgan and Stocken (2003) show that analysts' stock recommendations are asymmetrically distributed on "sell," "hold," and "buy." However, in their model analysts' biases are one-sided. Asymmetric equilibria are widespread in reality. Casual observation suggests there tends to be more "above average" recommendations than "below average" ones, in many situations of evaluations. In light of the result of this section, people may actually benefit from this seemingly uninformative recommendation scheme even if the underlying distributions are symmetric.

## Complete Information about Experts' Biases

With complete information about biases, simultaneous consultation allows full revelation of information, in equilibria constructed following Krishna and Morgan(2001a) (2001b) (see also Battaglini (2002)). When one of the expert is unbiased, the unbiased expert alone can perfectly reveal information. When both experts are biased, the decision maker's action can be specified such that self-serving messages are either ignored or punished. In particular, the decision maker interprets message  $m \in \{-1, 0, 1\}$  literally that the expert wants him to take action  $m$ . He follows the advice of the experts when the two agree. When experts disagree, there are two cases: the two experts have the same bias or opposite biases. For the former, the decision maker chooses to follow the advice that is self-hurting assuming the other expert's advice reveals the truth. For the latter, the decision maker adopts the following rules: if one of the experts recommends 0, then he always takes the extreme action that is opposite to the other expert's bias (e.g., 1 for a left-biased expert); if the two experts recommend exactly opposite actions, then he follows either one of them.

In sequential consultation, with complete information about biases, information can again be fully revealed if either expert is unbiased. If both experts are of the

same bias, however, they can reveal no more information than a single expert (see Krishna and Morgan (2001b)). The construction for simultaneous consultation does not work because in state 0 expert  $A$  will want to deviate from recommendation of 0 to her favored extreme action, knowing that expert  $B$  will concur, thereby inducing the decision maker to take the action. If they are of opposite biases, the construction in simultaneous consultation will not work either. To see this, suppose expert  $A$  has a right bias and expert  $B$  a left bias. Now, in state  $-1$ , when expert  $A$  deviates from recommendation of  $-1$  to 0, expert  $B$  cannot be relied upon to contradict  $A$ , because by so doing she brings about her least favored action, which is not sequentially rational. Therefore, full revelation is not achievable.

The above analysis implies that simultaneous consultation is the best mechanism with complete information about biases. My analysis on uncertainty about biases therefore provides fresh evidence that simultaneous consultation may be superior, even in the face of uncertainty about biases.<sup>11</sup> Knowing the experts' biases, the decision maker can ensure that a biased expert's contradiction never works in her favor, while when the expert's bias is not known, to achieve the same goal, the decision maker has to ignore the expert's report, thereby rendering the expert's report uninformative. This is because an action responsive to the expert's contradiction that is intended to punish a right-biased expert will reward a left-biased expert. Therefore, biased experts must "get their way" with positive probability to ensure informativeness of the other types of experts' reports.

It is straightforward to verify that, with complete information about the expert's bias, a single biased expert can reveal no information. Consequently, the decision maker prefers not to discover the expert's bias, even if it is costless to do so. Li and Madarasz (2008) show this to be true more generally when the decision maker is uncertain about the direction of an expert's bias, regardless of the degree of uncertainty. However, as simultaneous consultation enables full revelation of information, discovery of biases of *multiple* experts may still be desirable if the cost of discovery is not too high.

## 6 Related Literature

---

<sup>11</sup>See footnote 3, however, for a discussion of the limitation of this result.

This paper is closely related to research using the cheap talk model by Crawford and Sobel (1982) to study one decision maker obtaining advice from multiple experts.<sup>12</sup> Gilligan and Krehbiel (1989) and Krishna and Morgan (2001a) compare efficiency of the legislation process under “closed rule” and “open rule,”<sup>13</sup> when the legislature consults two perfectly informed committees (experts) with opposing biases on one piece of legislation. The former find that closed rule is better while the latter argue the opposite is true. Their conclusions are opposite due to their selection of different equilibria—Krishna and Morgan (2001a) show that the open rule can achieve full information revelation while the closed rule cannot.<sup>14</sup> Austen-Smith (1993) also studies legislation rules and compares sequential consultation and simultaneous consultation. In his model, the state and signal spaces are binary and the committees are imperfectly informed. He finds that sequential consultation is superior to simultaneous consultation. Krishna and Morgan (2001b) examine sequential consultation with two experts with like biases and opposing biases and conclude that the latter conveys more information than consultation a single expert, but the former does not. In light of results in their other paper (2001a), they argue that simultaneous consultation is superior to sequential consultation. Hori (2006) compares horizontal communication, hierarchical communication, and delegation in their ability to transmit information. In his model, information consists of two additive parts, each part observed perfectly by an expert. He finds that hierarchical communication dominates horizontal communication.

The main distinction between my paper and the above is that I allow biases of the experts to be unknown to the decision maker. Cheap talk with uncertainty about a single expert’s bias has been analyzed by Dimitrakas and Sarafidis (2005), Li and Madarasz (2008), Morgan and Stocken (2003), and Ottaviani (2000).

The “hierarchical review” mechanism I analyze in this model, where the reviewer decides what the decision maker receives from the expert, is related to the literature on mediated cheap talk, e.g., papers by Ganguly and Ray (2005), Mitusch and Strausz

---

<sup>12</sup>This paper is distinct from the literature of multidimensional cheap talk, where the information relevant to decision-making is multidimensional. See papers by Battaglini (2002) (2004), Ambrus and Takahashi (2008), Chakraborty and Harbaugh (2008), and Levy and Razin (2007). In my paper, though the expert has multidimensional private information, one of the dimensions, the expert’s bias, does not enter directly into the decision maker’s preferences.

<sup>13</sup>The open rule is the same as simultaneous consultation as in my model, while in the closed rule the second committee can only influence the legislature’s choice between the first committee’s proposal or a default status quo.

<sup>14</sup>However, their equilibrium construction relies on arguably implausible out-of-equilibrium beliefs. See Krehbiel (2001) and Battaglini (2002) for a detailed discussion.

(2005), and Goltsman, Horner, Pavlov, and Squintani (2008, forthcoming). However, the reviewer is equally informed as the expert, whereas the mediator is uninformed. In addition, the reviewer's strategies have to be incentive compatible, whereas the mediator is disinterested.

Another line of research incorporating uncertainty about the expert's type is concerned with how the expert's reputation concerns affect information transmission. Sobel (1985), Bénabou and Laroque (1992), Morris (2001), and Frisell and Lagerlöf (2007) focus on reputation for unbiasedness, Bourjade and Jullien (2004), Wei Li (2007), and Ottaviani and Sørensen (2006) focus on reputation for competence, and Olszewski (2004) focuses on reputation for honesty.

## 7 Concluding Remarks

In this paper, I have studied situations in which the decision maker consults multiple experts with uncertain biases. I compare three mechanisms: simultaneous consultation, sequential consultation, and hierarchical review. Simultaneous consultation does better than the other two mechanisms. Hierarchical review is a simple and less costly mechanism for the decision maker. It does achieve significant information transmission, sometimes better than sequential consultation.

In this paper, the state space and the expert type space are both discrete and the prior distributions are symmetric. However, it is also interesting to see investigate the case where all experts have biases in the same direction. Which mechanism brings about more effective communication?

Another extension is the possibility of reviewers and experts having different distribution of biases. Suppose there are two populations of experts with the same information, but different variances in bias. It is interesting to investigate which population should serve as reviewers, and which population as experts. In reality, decision makers may have better information about the reviewers' types, but it is not clear whether it is optimal to assign those who have lower variances in biases as reviewers.

## 8 Appendix: Proofs

In the following proofs, the reader is sometimes referred to the supplement to the paper, which can be found at [http://alcor.concordia.ca/~mingli/research/combsub\\_supp.pdf](http://alcor.concordia.ca/~mingli/research/combsub_supp.pdf).

*Proof of Proposition 1.* Since we consider symmetric equilibria,  $y_{-1} = -y_1$  and  $y_0 = 0$ . Let  $y = y_1$  to save notation. In informative equilibria,  $y > 0$ . Since  $y_s$  and  $s$  have the same sign, we have  $-(y_s - s)^2 < -(y_{s'} - s)^2$  for all  $s, s' \in S, s \neq s'$ . Therefore,  $m_0^*(s) = s$  for all  $s \in S$ .

Now, consider  $m_1^*(s)$ . Note that  $b_1 = b \in [17/21, 6/7]$ .

First, it is straightforward to see  $m_1^*(1) = 1$ . Second,  $m_1^*(0) = 1$ , as  $u(1, 0, b) = -(y - (0 + b))^2 > -(0 - (0 + b))^2 > -(-y - (0 + b))^2$  for all  $b > 1/2$  and  $y \in (0, 1]$ . Finally,  $m_1^*(-1) \neq 1$  because  $b < 1$  implies  $|0 - (-1 + b)| < |y - (-1 + b)|$ . If  $m_1^*(-1) = -1$  (by symmetry,  $m_{-1}^*(1) = 1$ ), then  $y = 3/4$ . Since  $b \geq 17/21 > 5/8$ ,  $|0 - (-1 + b)| = 1 - b < b - 1/4 = |-y - (-1 + b)|$ , which makes  $m_1(-1) = -1$  not optimal. If  $m_1^*(-1) = 0$ ,  $y = 2/3$ , thus  $|0 - (-1 + b)| = 1 - b < b - 1/3 = |-y - (-1 + b)|$  as  $b \geq 17/21 > 2/3$ . Hence,  $m_1^*(-1) = 0$  is optimal.

Thus, the strategy profile specified in the proposition is the unique symmetric equilibrium. Further, the decision maker's expected utility is  $-10/27$ , by straightforward calculation.  $\square$

*Proof of Proposition 2.* First, the following lemma describes the experts' strategies in equilibrium.

**Lemma 1.** *When the decision maker consults two experts simultaneously, in equilibrium, the following must be true about experts' strategies: for  $i = A, B$ ,  $m_0^i(s) = m_s^i(s) = s$  for  $s = -1$  and  $1$ , and  $m_0^i(0) = 0$ .*

The proof of it can be found in the supplement. Intuitively, unbiased experts never try to distort information. A biased expert tells the truth about the state when the state is at the extreme in the direction of her bias, as by so doing she induces her favorite action.

In what follows I only show that Strategy Profile (A) is an equilibrium. For the proof of uniqueness, please see the supplement to the paper.

Table 5 lists probabilities and decisions for each message pair.<sup>15</sup> Clearly  $m_1^A(0) = 1$  is optimal. I need also check the optimality of  $m_1^A(-1) = 0$ . I omit the calculation here, but as long as  $b > 58/105$ , it turns out for  $t = -1$  and  $1$

$$\frac{1}{3} \sum_{x \in X} u(y(0, m_x^A(0)), -1, b_1) > \frac{1}{3} \sum_{x \in X} u(y(t, m_x^A(0)), -1, b_1),$$

which is certainly satisfied since  $b \geq 17/21 > 58/105$ . Thus  $m_1^A(-1) = 0$  is optimal.

Straightforward calculation yields that the decision maker's expected payoff is  $-94/405$ .  $\square$

<sup>15</sup>The notation  $\text{Prob}(m^A, m^B, s = -1 + 0 + 1)$  means that in that column, probabilities of  $(m^A, m^B, s)$  are separated by "+" according to different  $s$ .

Table 5: Messages, probabilities, and decisions for simultaneous consultation

$(m^A, m^B)$	$\text{Prob}(m^A, m^B, s = -1 + 0 + 1)$	$y(m^A, m^B)$
(1, 1)	$0 + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3}$	$\frac{4}{5}$
(1, 0) or (0, 1)	$0 + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}$	$\frac{2}{3}$
(1, -1)	$0 + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + 0$	0
(0, 0)	$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$	0

*Proof of Proposition 3.* I first establish a lemma that describes experts' equilibrium strategies, the proof of which can be found in the supplement to the paper.

**Lemma 2.** *When the decision maker consults two experts sequentially, the following must be true about experts' strategies in equilibrium:*

1.  $m_0^A(1) = m_1^A(1) = 1$  and  $m_0^A(0) = 0$ ;
2.  $m_0^B(1, m^A), m_1^B(1, m^A) \in \arg \max_{m^B} y(m^A, m^B)$ ;
3.  $m_0^B(0, 0) = 0, m_1^B(0, -1) \in \arg \max_{m^B} y(-1, m^B)$ , and  $m_1^B(0, 0) \in \arg \max_{m^B} y(0, m^B)$ .

Monotonicity requires that expert  $A$  reports 1 in state 1 if she is of type 1. If she does not, then monotonicity implies  $m_x(s) = 0$  for all  $x \in X$ , which renders expert  $A$ 's reports uninformative. Symmetry requires that the first expert reports 0 in state 0 when she is of type 0. When the true state is 1 and the expert is of type 1 or 0, the expert wants the decision maker to take the highest action. The other results in the lemma follow similar lines of argument.

By Lemma 2, the only parts of expert  $A$ 's strategy left to be determined are  $m_1^A(-1)$  and  $m_1^A(0)$ . Note that in strategy profile (C),  $m_1^A(-1) = 0$  and  $m_1^A(0) = 1$ . Here, I only verify that strategy profile (C) is an equilibrium. Please refer to the supplement of the paper for the argument that it is the most informative among all monotonic symmetric equilibria.

Note that in strategy profile (C),  $y(0, -1) < y(0, 0)$ . This immediately implies  $m_1^B(0, 0) = 1, m_1^B(1, 0) = 1$ , and  $m_0^B(1, 0) = 1$ . We also know  $m_1^B(-1, 0) \neq 1$  since  $y(0, 0)$  is better than  $y(0, 1)$  in state  $-1$  for an expert of type 1. Since  $y(0, 1) \geq 2/3$ , we conclude that  $m_1^B(-1, 0) = 0$  as a type 1 expert's most preferred action in state  $-1$  is  $-1 + b > -1/3$ , which is closer to  $y(0, 0) = 0$  than to  $y(0, -1)$ . This gives us  $y(0, 1) = 2/3$ .

If  $y(1, 1) = y(1, 0)$ , it is possible that (1, 1) or (1, 0) (but not both) is never sent, but it does not matter to our discussion since we may replace them with each

other without essentially changing the strategy profile. Now we have  $y(1, 1) > 2/3 > y(1, -1)$ , which implies  $m_{-1}^B(1, 1) = -1$  and  $m_{-1}^B(0, 1) = m_0^B(0, 1) = -1$  as  $b \geq 2/3$ . Hence  $y(1, -1) \leq 1/2$ , which implies  $m_1^B(0, 1) \neq -1$  since  $b \geq 17/21 > 3/4$  and  $y(1, 1) \leq 1$ . Thus,  $y(1, 1) = y(1, 0) = 4/5$  and  $y(1, -1) = 1/2$ . The case  $y(1, 0) = y(1, -1)$  is similar, which results in  $y(1, 1) = 4/5$  and  $y(1, 0) = y(1, -1) = 1/2$ .

Now, I check the optimality of  $m_1^A(-1) = 0$  and  $m_1^A(0) = 1$ . First, note that  $m_1^A(-1) = 0$  induces actions  $y(0, -1) = -2/3$  with probability  $2/3$  and  $y(0, 0)$  with probability  $1/3$ , that  $m_1^A(-1) = -1$  induces actions  $y(-1, -1) = -4/5$  with probability  $2/3$  and  $y(-1, 1) = -1/2$  with probability  $1/3$ , and that  $m_1^A(-1) = 1$  induces action  $y(1, -1) = 1/2$  for sure. Thus  $m_1^A(-1) = 0$  is better than  $m_1^A(-1) = -1$  since a type 1 expert prefers  $-2/3$  to  $-4/5$  and  $0$  to  $-1/2$  in state  $-1$ , due to our assumption  $b \in [17/21, 6/7]$ . The difference in expected utility between  $m_1^A(-1) = 0$  and  $m_1^A(-1) = 1$  is

$$-\frac{1}{3}[2(-\frac{2}{3} - (-1 + b))^2 + (0 - (-1 + b))^2] - (\frac{1}{2} - (-1 + b))^2 = \frac{199}{36} - \frac{17}{3}b,$$

which is positive as long as  $b < 199/204$ . Thus,  $m_1^A(-1) = 0$  is better than  $m_1^A(-1) = 1$  given our assumptions. Second,  $m_1^A(0) = 1$  induces actions  $y(1, 1) = 4/5$  with probability  $1/3$  and  $y(1, -1) = 1/2$  with probability  $2/3$ , while  $m_1^A(0) = 0$  induces actions  $y(0, 1) = 2/3$ ,  $y(0, 0) = 0$ , and  $y(0, -1) = -2/3$  with equal probabilities. Thus  $m_1^A(0) = 1$  is a better response than  $m_1^A(0) = 0$  since a type 1 expert prefers  $4/5$  to  $2/3$  and  $1/2$  to any nonpositive action, due to our assumption  $b \in [17/21, 6/7]$ . Thus, the strategy profile constitutes an equilibrium. Note that it is **strategy profile (C)**. In this strategy profile, it is straightforward to show the decision maker's expected payoff is  $-104/405$ .  $\square$

*Proof of Proposition 4.* First, I establish two lemmas regarding the reviewer and the expert's equilibrium strategies.

If a reviewer of a certain type rejects a message in some state, then intuitively, an expert of the same type should never send that message in the same state, as there must exist an alternative message that she strictly prefers. The following lemma establishes this fact.

**Lemma 3.** *In equilibrium, if  $r_v^*(s, t) = 1$ , then  $\tilde{m}_v^*(s, t) = 0$ , i.e.,  $m_v^*(s) \neq t$ .*

*Proof.* In equilibrium, a reviewer of type  $v \in X$  rejects message  $t \in M$  in state  $s \in S$  if and only if

$$\sum_{t'} -\gamma_{t'} [y_{t'}^* - (s + b_v)]^2 > -[y_t^* - (s + b_v)]^2. \quad (2)$$

Since  $r_v^*(s, t) = 1$ , Equation (2) must hold. This implies that  $-[y_t^* - (s + b_v)]^2 = u(y_t^*, s, b_v) < \max_{t' \in S} u(y_{t'}^*, s, b_v)$ . Let  $\tilde{t} = \arg \max_{t' \in S} u(y_{t'}^*, s, b_v)$ . Then if an expert of type  $v$  sends the message  $\tilde{t}$  in state  $s$ , her expected payoff is at least  $\sum_{t'} \gamma_{t'} u(y_{t'}^*, s, b_v)$ , which is greater than  $u(y_t^*, s, b_v)$  by Equation (2). Since the expert's expected payoff from sending message  $t$  is a convex combination of this expression and  $u(y_t^*, s, b_v)$ , the expert is strictly better off sending message  $\tilde{t}$ . Hence  $\tilde{m}_v^*(s, t) = 0$ .  $\square$

Note that I have not used symmetry in the above proof. So, Lemma 3 applies to all equilibria of the game, not just symmetric ones. Now, I establish that the only possible behavior of the reviewer in symmetric equilibria is that in (E2), which also implies certain behavior of the expert.

**Lemma 4.** *In a symmetric equilibrium,*

1.  $r_0^*(s, t) = 1$  if  $(s, t) = (-1, 1), (0, -1), (0, 1)$  or  $(1, -1)$ , and 0 otherwise;  $r_1^*(s, t) = 1$  if  $(s, t) = (-1, 1), (0, -1)$  or  $(1, -1)$ , and 0 otherwise;
2.  $m_0(s) = s$  for all  $s \in S$  and  $m_1(1) = 1$ .

The proof of the above lemma can be found in the supplement to the paper. Intuitively, rejection happens only if the reviewer prefers the resulting random message drawn from the endogenous symmetric distribution to the original message. Because of the quadratic loss utility function, no expert/reviewer prefers the random message to the message 0. Therefore, a reviewer never rejects the message 0. At the same time, a reviewer of type 0 rejects messages 1 and  $-1$  in state 0 and message 1 in state  $-1$ , since these messages are the worst for her to pass on to the decision maker. For similar reasons, a reviewer of type 1 rejects the message  $-1$  when the state is 0 or 1. These arguments do not depend on the size of the bias (as long as  $b > 1/2$ , which precludes a fully revealing equilibrium). However, the argument for type 1 not rejecting  $-1$  in state  $-1$  and rejecting 1 in state  $-1$  does depend on the fact that her bias is not very large.

Now we resume the proof of Proposition 4. By Lemmas 3 and 4,  $m_1(-1) \neq 1$  and  $m_1(0) \neq -1$ . What is left to be determined is whether  $m_1(0) = 0$  or 1 and whether  $m_1(-1) = -1$  or 0. Observe that since when the decision maker receives no messages, a signal is randomly drawn from the endogenously generated distribution

$(\gamma, 1 - 2\gamma, \gamma)$ , the following must be true:

$$\begin{aligned}\gamma &= P(s = 1)[P(x = 0, 1) + P(x = 1)(1 - \tilde{m}_1(-1, 0))] \\ &\quad + P(s = 0)P(x = 1)\tilde{m}_1(0, 1)[P(v = 1) + P(v = -1, 0)\gamma] \\ &\quad + P(s = 0)P(x = -1)\tilde{m}_{-1}(0, -1)P(v = 0, 1)\gamma \\ &= \left(\frac{1}{3} - \frac{\tilde{m}_1(-1, 0)}{9}\right) + \frac{\tilde{m}_1(0, 1)}{9}\left(1 - \frac{2}{3}(1 - 2\gamma)\right)\end{aligned}$$

I used the fact  $\tilde{m}_{-1}(0, -1) = \tilde{m}_1(0, 1)$  by symmetry. From the above equation I get

$$(\dagger) \quad \gamma = \frac{1}{9}\left(1 - \frac{2}{3}(1 - 2\gamma)\right)\tilde{m}_1(0, 1) - \frac{\tilde{m}_1(-1, 0)}{9} + \frac{1}{3}.$$

Note that

$$(\dagger\dagger) \quad y_1 = \frac{P(s=1, m=1)}{P(m=1)} = \frac{1}{\gamma} \left[ \frac{1}{3} - \frac{\tilde{m}_1(-1, 0)}{9} \right].$$

Given the reviewer's strategies, the decision of an expert of type 1 should be characterized by the following comparisons.

- (i) Since neither 0 nor  $-1$  is ever rejected in state  $-1$ ,  $m_1(-1) = 0$  if

$$y_1 \geq 2(1 - b).$$

- (ii) Since in state 0, 0 is not rejected and 1 is rejected by types 0 and  $-1$ , we get  $m_1(0) = 0$  is optimal if

$$\begin{aligned}-b^2 + \left[\left(1 - \frac{2}{3}\right)(y_1 - b)^2 + \frac{2}{3}(2\gamma y_1^2 + b^2)\right] &\geq 0 \\ \Leftrightarrow y_1 &\geq \frac{2b\left(1 - \frac{2}{3}\right)}{1 - \frac{2}{3}(1 - 2\gamma)}\end{aligned}$$

- (iii) Using (2), in order for  $r_1(-1, -1) = 0$  to be optimal we need

$$y_1 \leq \frac{2(1 - b)}{1 - 2\gamma}.$$

Since there exist no fully revealing equilibria, we only need to consider three cases:

- (a)  $m_1(-1) = 0$  and  $m_1(0) = 0$ ; (b)  $m_1(-1) = 0$  and  $m_1(0) = 1$ ; (c)  $m_1(-1) = -1$  and  $m_1(0) = 1$ .

- (a) It can also be represented as  $\tilde{m}_1(-1, 0) = 1$  and  $\tilde{m}_1(0, 1) = 0$ .

By  $(\dagger)$ ,  $\gamma = 2/9$ .

By  $(\dagger\dagger)$  we have  $y_1 = (1/3 - 1/9)/(2/9) = 1$ .

According to condition (iii) in this proof, we need  $1 \leq 2(1 - b)/(5/9)$ . This requires  $b \leq 13/18$ , which does not hold given the assumption  $b \in [17/21, 6/7]$ .

(b) This case can be represented by  $\tilde{m}_1(-1, 0) = 1$  and  $\tilde{m}_1(0, 1) = 1$ .

By Equation (†),  $\gamma = \frac{2/9+1/9 \cdot 1/3}{1-2/9 \cdot 2/3}$ .

By Equation (††),  $y_1 = \frac{2}{9}/\gamma$ .

According to condition (i) in this proof, we need  $y_1 \geq 2(1-b)$ , which translates into  $(1-b-2/9)(1-2/3) \leq 1-2(1-b)-2/9$ . This holds since  $b \geq 2(1-b)$  for our assumed values of  $b$ .

In this case  $1-2/3 \cdot (1-2\gamma) = 9(\gamma-2/9)$ . Substituting this into condition (ii) in this proof, we get  $9y_1(\gamma-2/9) \leq 2b \cdot (1-2/3)$ . Using  $\gamma y = 2/9$ , we get

$$b(1-\frac{2}{3})^2 + (2b-\frac{5}{9})(1-\frac{2}{3}) - \frac{4}{9} \geq 0,$$

which is equivalent to  $\frac{7}{9}b - \frac{17}{27} \geq 0$  or  $b \geq \frac{17}{21}$ . By Assumption 1, condition (ii) is satisfied.

Now we check condition (iii). We need  $2/9 \cdot (1-2\gamma/\gamma) \leq 2(1-b)$ , which simplifies into

$$1 - \frac{2}{3} \geq \frac{1}{3(1-b)} - 2.$$

This inequality holds when  $b \leq 6/7$ , which is satisfied by Assumption 1. So we conclude that **strategy profile (E)** is an equilibrium.

(c) This case can be represented by  $\tilde{m}_1(-1, 0) = 0$  and  $\tilde{m}_1(0, 1) = 1$ .

By Equation (†),  $\gamma = \frac{1/3+1/9 \cdot (1-2/3)}{1-2/9 \cdot 2/3}$ .

By Equation (††),  $y_1 = \frac{1}{3}/\gamma$ .

By condition (i) of this proof, we need  $y_1 \leq 2(1-b)$ , which requires  $13 \cdot (7/9 + 2/9 \cdot (1-2/3)) \leq (1-b)(2/3 + 2/9 \cdot (1-2/3))$ , an impossible statement since  $1-b < 1/3$ . So this is not an equilibrium strategy profile.

Summarizing the above arguments proves the proposition. □

*Sketch of Proof of Proposition 5.* First, I establish a lemma that applies to all equilibria of the hierarchical review mechanism.

**Lemma 5.** *In equilibrium, the following must be true for all  $v, v' \in X$ ,  $s \in S$ , and  $t \in M$ , where  $v \geq v'$ :*

1. *If  $y_t > 0$  then  $r_v(s, t) = 1$  implies  $r_{v'}(s, t) = 1$ ;*
2. *If  $y_t < 0$  then  $r_v(s, t) = 1$  implies  $r_{v'}(s, t) = 1$ ;*
3. *If  $y_t = 0$  then  $r_v(s, t) = 0$ .*

The proof of the above lemma can be found in the supplement. Intuitively, a left-biased reviewer is more likely to reject a message that induces a positive action, compared with neutral and right-biased reviewers. Note also that the same statement can be stated in terms of acceptance instead of rejection, in reverse order. This lemma is useful since it implies that at most two review decisions need to be checked for any given state-message pair.

A fact worth noting is that  $\gamma_1 y_1 = \frac{2}{9}$  and  $\gamma_{-1} y_{-1} = -\frac{1}{3}$ . Now I check the optimality of the reviewer's decisions. Consider any  $r_v(s, t)$ , where  $v \in X$ ,  $s \in S$ , and  $t \in M$ . Observe that the difference in utility between rejection and acceptance is

$$-\left[\sum_{t'} \gamma_{t'} (y_{t'} - (s + b_v))^2 - (y_t - (s + b_v))^2\right] = -\left[\sum_{t'} \gamma_{t'} y_{t'}^2 - y_t^2 + 2y_t(s + b_v)\right].$$

Many results can be obtained by using Lemma 5. Here, I only consider the optimality of  $r_{-1}(1, 1) = 0$ , which is crucial to the proposition and the proof of other review decisions follow similar procedures. The difference in utility between rejection and acceptance is

$$-y_1 \left[ \frac{2}{9} + \frac{1}{9} \cdot \frac{3}{7} + \frac{1}{3} \cdot \frac{9}{10} - \frac{23}{27} + 2(1 - b) \right],$$

which is negative as long as  $b < 3247/3780$ . This is satisfied as by Assumption 1,  $b \leq 6/7 = 324/378 < 3247/3780$ . That  $r_0(1, 1) = r_1(1, 1) = 0$  comes from the fact that  $y_1$  is the reviewer's most preferred action in state 1.

Finally, I check the optimality of the expert's reports. First,  $m_0(1) = m_1(1) = 1$ ,  $m_0(-1) = m_{-1}(-1) = -1$ , and  $m_{-1}(1) = 0$  since the messages correspond to the expert's most preferred action, and are never rejected. Second, by Lemma 3,  $m_{-1}(0) = -1$  and  $m_0(0) = 0$  since a reviewer of the same type rejects the other two available messages. Last, I show that  $m_1(0) = 0$  and  $m_1(-1) = -1$  are optimal. As a first step,  $m_1(0) \neq -1$  and  $m_1(-1) \neq 1$  due to Lemma 3 and the fact that  $r_1(0, -1) = r_1(-1, 1) = 1$ . Note that  $y_1 = 23/27$ ,  $y_0 = \frac{3}{7}y_1$ , and  $y_{-1} = -\frac{9}{10}y_1$ . In state 0, for an expert of type 1, the difference in utility between sending 0 and sending 1 is

$$\begin{aligned} & \frac{1}{3}[(y_1 - b)^2 + \sum_m \gamma_m (y_m - b)^2 - 2(y_0 - b)^2] \\ &= \frac{1}{3}y_1 \left[ \frac{31}{49}y_1 - \frac{2}{7}b + \frac{2}{9} + \frac{1}{21} + \frac{3}{10} \right] > 0, \end{aligned}$$

using the facts  $\sum_m \gamma_m y_m = 0$  and  $b \leq 1$ . Thus  $m_1(0) = 0$  is optimal.

In state 1, following similar procedures to that of the previous paragraph, I obtain that the difference in utility for an expert of type 1 between sending  $-1$  and 0 in state  $-1$  is

$$\frac{1}{3}y_1 \left[ \left( \frac{9}{49} - \frac{243}{100} \right) y_1 + \left( \frac{6}{7} + \frac{27}{5} \right) (1 - b) + \frac{4}{9} + \frac{2}{21} + \frac{3}{5} \right] > 0.$$

The inequality is gotten by substituting  $y_1 = \frac{23}{27}$  and using my assumption that  $b \leq \frac{6}{7}$ . Hence  $m_1(-1) = -1$  is optimal.

Thus, strategy profile (F) constitutes an equilibrium. The decision maker's payoff under the asymmetric equilibrium as prescribed by Proposition 5 is

$$-\frac{2}{3} + \sum_m \gamma_m y_m^2 = -\frac{2}{3} + \left(1 - \frac{4}{27}\right) \left(\frac{2}{9} + \frac{1}{9} \cdot \frac{\frac{1}{9}}{\frac{1}{9} + \frac{2}{9} \cdot \frac{2}{3}} - \frac{1}{3} \cdot \frac{-\frac{1}{3}}{\frac{1}{3} + \frac{1}{9} \cdot \frac{1}{3}}\right) = -\frac{3083}{17010},$$

which is higher than his payoff  $-194/567 = -5820/17010$  under the symmetric equilibrium.  $\square$

## References

- ADMATI, A. R., AND P. PFLEIDERER (2004): "Broadcasting opinions with an overconfident sender," *International Economic Review*, 45(2), 467–497.
- AMBRUS, A., AND S. TAKAHASHI (2008): "Multi-sender cheap talk with restricted state spaces," *Theoretical Economics*, 3(1), 1–27.
- AUSTEN-SMITH, D. (1993): "Interested Experts and Policy Advice: Multiple Referrals under Open Rule," *Games and Economic Behavior*, 5(1), 1–43.
- BATTAGLINI, M. (2002): "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica*, 70(4), 1379–1401.
- (2004): "Policy Advice with Imperfectly Informed Experts," *Advances in Theoretical Economics*, 4(1), Article 1.
- BÉNABOU, R., AND G. LAROQUE (1992): "Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility," *Quarterly Journal of Economics*, 107(3), 921–958.
- BLUME, A., O. BOARD, AND K. KAWAMURA (2007): "Noisy Talk," *Theoretical Economics*, 2, 395–440.
- BOARD, O., AND T. DRAGU (2006): "Expert Advice with Multiple Decision Makers," *working paper*.
- BOURJADE, S., AND B. JULLIEN (2004): "Expertise and Bias in Decision Making," MPRA Paper 7251, University Library of Munich, Germany.
- CHAKRABORTY, A., AND R. HARBAUGH (2008): "Clearly biased experts," *mimeo*.
- CHEN, Y., N. KARTIK, AND J. SOBEL (2008): "Selecting Cheap-Talk Equilibria," *Econometrica*, 76(1), 117–136.
- CRAWFORD, V., AND J. SOBEL (1982): "Strategic Information Transmission," *Econometrica*, 50(6), 1431–1452.

- DESSEIN, W. (2002): “Authority and Communication in Organizations,” *Review of Economic Studies*, 69, 811–838.
- DIMITRAKAS, V., AND Y. SARAFIDIS (2005): “Advice from an Expert with Unknown Motives,” *mimeo*, INSEAD.
- FARRELL, J. (1993): “Meaning and credibility in cheap-talk games,” *Games and Economic Behavior*, 5, 514–31.
- FARRELL, J., AND M. RABIN (1996): “Cheap Talk,” *Journal of Economic Perspectives*, 10(3), 103–118.
- FRISELL, L., AND J. N. M. LAGERLÖF (2007): “A Model of Reputation in Cheap Talk,” *Scandinavian Journal of Economics*, 109(1), 49–70.
- GANGULY, C., AND I. RAY (2005): “Can Mediation Improve Upon Cheap-Talk? A Note,” Discussion Papers 05-08, Department of Economics, University of Birmingham.
- GILLIGAN, T., AND K. KREHBIEL (1987): “Collective Decision-Making and Standing Committees: An Informational Rationale for Restrictive Amendment Procedures,” *Journal of Law, Economics and Organization*, 3, 287–335.
- (1989): “Asymmetric Information and Legislative Rules with a Heterogeneous Committee,” *American Journal of Political Science*, 33(2), 459–490.
- GOLTSMAN, M., J. HORNER, G. PAVLOV, AND F. SQUINTANI (2008, forthcoming): “Arbitration, Mediation, and Cheap Talk,” *Journal of Economic Theory*.
- HORI, K. (2006): “Organizing Information Transmission with Multiple Agents,” *mimeo*.
- KREHBIEL, K. (2001): “Plausibility of Signals by a Heterogeneous Committee,” *American Political Science Review*, 95(2), 453–457.
- KRISHNA, V., AND J. MORGAN (2001a): “Asymmetric Information and Legislative Rules: Some Amendments,” *American Political Science Review*, 95(2), 435–452.
- (2001b): “A Model of Expertise,” *Quarterly Journal of Economics*, 116(2), 747–775.
- LEVY, G., AND R. RAZIN (2007): “On the Limits of Communication in Multidimensional Cheap Talk: A Comment,” *Econometrica*, 75(3), 885–893.
- LI, M. (2008): “Two (talking) heads are not better than one,” *Economics Bulletin*, 3(63), 1–8.
- LI, M., AND K. MADARASZ (2008): “When mandatory disclosure hurts: Expert advice and conflicting interests,” *Journal of Economic Theory*, 139(1), 47–74.

- LI, W. (2007): “Changing Ones Mind when the Facts Change: Incentives of Experts and the Design of Reporting Protocols,” *Review of Economic Studies*, 74, 1175–194.
- MATTHEWS, S. A., M. OKUNO-FUJIWARA, AND A. POSTLEWAITE (1991): “Refining Cheap-Talk Equilibria,” *Journal of Economic Theory*, 55(2), 247–273.
- MITUSCH, K., AND R. STRAUZ (2005): “Mediation in Situations of Conflict and Limited Commitment,” *Journal of Law, Economics and Organization*, 21(2), 467–500.
- MORGAN, J., AND P. STOCKEN (2003): “An Analysis of Stock Recommendations,” *Rand Journal of Economics*, 34(1), 183–203.
- MORRIS, S. (2001): “Political Correctness,” *Journal of Political Economy*, 109, 231–265.
- MYLOVANOV, T. (2008): “Veto-Based Delegation,” *Journal of Economic Theory*, 138(1), 297–307.
- OLSZEWSKI, W. (2004): “Informal communication,” *Journal of Economic Theory*, 117(2), 180–200.
- OTTAVIANI, M. (2000): “The Economics of Advice,” *working paper, University College London*.
- OTTAVIANI, M., AND P. N. SØRENSEN (2006): “Professional Advice,” *Journal of Economic Theory*, 126(1), 120–142.
- SOBEL, J. (1985): “A Theory of Credibility,” *Review of Economic Studies*, 52(4), 557–573.