ECON 421/521: Midterm Exam February 16, 2004

Read the questions carefully. Write clear and complete answers. Show your work to get credit. Partial answers get partial credit.

Question 1 (24 points). Suppose that the true wage equation is given by

$$\ln(wage) = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 IQ + \beta_4 exper^2 + \beta_5 educ.exper + u \tag{1}$$

where *wage* is the hourly wage, *educ* is years of schooling, *exper* is years of experience and *IQ* is intelligence test score in points. The OLS estimation of the model, using a random sample of individuals, gives the following results

$$\begin{aligned} \ln(\widehat{wage}) &= \underset{(.171)}{-.233} + \underset{(.012)}{.117}\ educ + \underset{(.009)}{.062}\ exper + \underset{(0.0010)}{0.0059}\ IQ - \underset{(.0001)}{.0008}\ exper^2 - \underset{(.0005)}{.0014}\ educ.exper \quad (2) \\ n &= 526,\ R^2 = 0.35,\ SSE = 148 \end{aligned}$$

(a) Interpret the coefficient on IQ. Why is IQ included in the model? If IQ increases by 20, what is the percentage change in the wage? Construct 95% confidence interval (critical value=1.96) for the coefficient on IQ. Use the computed confidence interval to test the hypothesis that IQ has no ceteris paribus effect on $\ln(wage)$.

(b) What is the percentage return to experience of an individual with 10 years of schooling and 5 years of experience? What is the percentage return to education of an individual with 5 years of experience?

(c) Write down the null hypothesis that the return to education of an individual with 5 years of experience is zero. Describe briefly how you would test this hypothesis using a t-test and the information from a standard regression output.

(d) Use the information provided in (2) to compute the standard error of the regression, $\widehat{\sigma}$.

Question 2 (36 points). The true (population) model is given by

$$y = \beta_1 x_1 + \beta_2 x_2 + u, \tag{3}$$

where $\{y_i, x_{i1}, x_{i2}\}_{i=1}^n$ is a random sample, $E(u|x_1, x_2) = 0$ and x_1 and x_2 are not perfectly collinear.

Suppose that instead of (3) you decided to estimate by OLS the model

$$y = \beta_1 x_1^* + \beta_2 x_2 + e, \tag{4}$$

where x_1^* is the OLS residual from a regression of x_1 on a constant and x_2. Let $\widehat{\beta}_1$ and $\widehat{\beta}_2$ denote the OLS estimators from model (3) and $\widetilde{\beta}_1$ and $\widetilde{\beta}_2$ denote the OLS estimators from model (4).

(a) Show that the estimator $\widetilde{\beta}_2$ of β_2 from (4) is identical to the regression coefficient from a simple regression of y on x_2.

(b) Use the result from part (a) to obtain an expression for the bias of the estimator $\widetilde{\beta}_2$. Does the bias arise from omitting an important variable? Explain.

(c) Prove that the estimators $\widehat{\beta}_1$ and $\widetilde{\beta}_1$ are identical (Hint: Use the "partialling out" (two-step) approach that we discussed in class).