ECON 421/521, Fall 2009
Assignment 4
Due Date: Monday, December 7

Problems: **10.2** from Chapter 10 and **11.4** from Chapter 11 (*Introductory Econometrics* by J. Wooldridge)

Computer Exercises: **C10.10** from Chapter 10 and **C11.8** (without part (iv)) from Chapter 11 (10.16 and 11.15 in the previous editions).
The data files for the computer exercises are:

- `INTDEF.RAW`: 56 observations with 4 columns: `year`, `i3`, `inf` and `def` (the variables are described in Example 10.2)
- `PHILLIPS.RAW`: 56 observations with 3 columns: `year`, `unem` and `inf` (annual US unemployment and inflation rates from 1948 to 2003. The actual unemployment rate for 2004 is 5.5)

and can be downloaded from `http://alcor.concordia.ca/~gospodin/teaching/421/421.html`
Some STATA operators for time series analysis:

- To use the time series operators, you must declare that the data are time series by typing

  `tsset year`

- To create the first and second lags of variable `x`, type

  `gen x1=L.x`

  `gen x2=L2.x`

- To compute the correlation between `x` and `y`, type

  `correlate x y`

Additional Problem 1. Suppose that the time series model

$$y_t = \beta_0 + \beta_1 x_t + u_t \qquad (1)$$

satisfies assumptions $TS.1'$ (linearity and weak dependence) and $TS.2'$ (no perfect collinearity) in the text.
Consider the following three specifications of the model and answer the questions.

(a) $E(u_t|x_t) = 0, Var(u_t|x_t) = \sigma_u^2 x_t^2$ and $E(u_t u_s|x_t, x_s) \neq 0$ for $s \neq t$. Is the OLS estimator in (1) unbiased? Why? Is the OLS estimator in (1) consistent? Why? Is the OLS estimator BLUE?

(b) $E(u_t|x_t) = 0, E(u_t u_s|x_t, x_s) = 0$ for $s \neq t$ and $Var(u_t|x_t) = h_t$, where $h_t = \exp(\gamma_0 + \gamma_1 x_t^2 + \gamma_2 \frac{1}{x_t^4})$ but the values of γ_0, γ_1 and γ_2 are unknown. Can we estimate the variance function by OLS? Show how we can obtain a consistent estimator of β_1 that is more efficient than the OLS estimator (show all the steps).

(c) $Var(u_t|x_t) = \sigma_u^2, E(u_t u_s|x_t, x_s) = 0$ for $s \neq t$ and $x_t = \alpha_0 + \alpha_1 y_t + e_t$. Is the OLS estimator in (1) consistent? If yes, explain why. If no, propose a consistent estimator of β_1.

Additional Problem 2.

(a) Suppose that the time series process y_t is generated by

$$y_t = e_t + \theta e_{t-1},$$

where θ is a finite constant and $e_t \sim iid(0, \sigma^2)$ with $\sigma^2 < \infty$. Show if the process y_t is covariance (weakly) stationary and weakly dependent.

(Hint: Recall that the covariance stationarity puts some restrictions on the mean, the variance and the covariance of the process and the weak dependence imposes some additional conditions on the correlation between two realizations of the process).

(b) Consider the model

$$\begin{aligned} y_t &= \beta y_{t-1} + u_t \qquad (2) \\ u_t &= \rho_1 u_{t-1} + \rho_2 u_{t-2} + e_t, \end{aligned}$$

where $e_t \sim iid(0, \sigma^2)$. Show that the process for y_t can be rewritten as $y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + e_t$, where α_1, α_2 and α_3 are functions of β, ρ_1 and ρ_2. Discuss the consequences on the estimator of β if model (2) is estimated by OLS.

(c) Consider the FDL(2) model

$$y_t = \delta_0 x_t + \delta_1 x_{t-1} + \delta_2 x_{t-2} + u_t, \qquad (3)$$

where u_t is serially uncorrelated. Rewrite the model (3) in a form that would allow you to test directly the hypothesis that the long-run propensity (multiplier) is equal to 1.